Note on Determining the Fair Value of a Variable Rate Callable Loan

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Introduction

It is well accepted that the value of a callable variable rate debt security is equal to its value without the call feature (straight security) less the value of the call feature. Although bonds and loans are debt securities there are notable differences. Loans are generally senior to bonds in the cap stack, have coupons that are typically variable, often with LIBOR floors, and loans are callable at any time over their life while bonds typically have a fixed time frame within which the issuer can call the security, e.g. anytime within the first two years after issuance. While in both the callable bond and loan cases the investors face prepayment risk, this risk is generally greater for the investor in the loan than in the callable bond. This note addresses the valuation of variable rate callable loans and specifically focuses on the value of the call embedded in these loans.

The reason for the focus on the embedded call option is because loans appear to trade at or below par even when the coupon is far larger than the loan's required rate of return based on credit risk. This implies that the embedded call is valuable to the issuer and costly for the investor. There is little academic literature that addresses this issue while anecdotal evidence seems to indicate that professional investors in the syndicated and leveraged loan markets believe that pre-payment risk is very high thus offsetting a percentage of the value investors would receive when coupon rates are far larger than the yield based on the security's credit risk. This fact pattern then gives rise to the question under what conditions would a call be activated and subsequently exercised. The model described herein sets out a methodology for valuing the embedded call thus enabling a more precise calculation of the fair value of the underlying variable rate loan.

Setting the Stage

Call features on loans are triggered by various events. When a change in control transaction takes place, loans as well as other liabilities are typically paid off with proceeds of the acquisition. This is a non-systematic call trigger because take-over transactions tend to be episodic and are more often than not unpredictable. However, during periods when acquisition frequency within an industry increases, the likelihood that affected loans will be called increases. While the timing of acquisition related calls are difficult to determine, such activity increases prepayment risk above what traditional factors would normally dictate with the upshot being that the resulting call values are larger and the value of the loans lower than under more normal market conditions. The methodology described in this note can easily accommodate an acquisition scenario, but because such a scenario adds substantive complications, we have chosen not to address it herein but rather to focus on the two fundamental variables that drive the value of a variable rate callable loan.

The table below summarizes the non- change in control events that can trigger a call.

Table 1-1: Call Conditions

	Credit Spread Narrowed as of Measurement Date	Credit Spread Unchanged in the Future	Credit Spread Widened in the Future
Credit Rating Unchanged as of Measurement Date	Call	No Call	No Call
Credit Rating Improved as of Measurement Date	Call	Call	Maybe
Credit Rating Improved in the Future	Call	Call	Maybe

As the table above indicates, the call is always activated once the market credit spread falls below the loan's contractual coupon spread. We refer to this difference as the credit spread-coupon spread differential or CCSD. The call will be triggered if the savings from refinancing exceeds the call premium plus fees associated with refinancing. In cases where the coupon spread and market credit spread are equal, the call will be activated whenever there is an improvement in the loan's credit rating which typically means that the credit rating of the issuer has improved. However, if the issuer's credit rating improves at some future point and the new coupon spread at the higher rating is not lower than the contractual coupon spread in the existing loan, then, despite the improved credit rating, the loan will not be called. The loan will only be called if the new coupon spread is sufficiently below the contractual coupon spread of the current loan such that the savings in coupon interest is greater than the costs associated with refinancing.

Let us consider this last fact pattern in more depth since it encompasses a number of important complications. These include being able to project both credit spreads and changes in the firm's credit rating over the life of the loan. Projecting credit spreads is straightforward. First, we calculate the term structure of Treasury rates at the measurement date. We then calculate the term structure of interest rates for each credit risk level from AAA+ to D. Under normal market conditions the term structure of interest rates is upward slopping. This implies, for example, that the 5 year BB+ rate is larger one year from the measurement date than it is at the measurement date. By appealing to expectations' theory of the term structure, one can solve for all future rates across the risk spectrum over the life of the loan. Projected future credit spreads are simply the difference between the projected rate for a given risk class at a given maturity less the projected risk free rate with the equivalent maturity.

In comparison to projecting expected future credit spreads, projecting the firm's credit risk and the credit risk assigned to the loan over the life of the loan is far more complicated. The reason is that in order to do this one needs a credit model in addition to being able to project the inputs needed by the model in order to project the degree to which credit risk changes over the life of the loan. In the example used here, we use Axiom's credit rating platform to measure changes in credit risk.

Methodology

The methodology used to price a variable rate callable loan is divided into four discrete steps.

1. Step 1: Calculate the loan's YTM based on its credit risk at the measurement date. Based on this and the contractual cash flows, calculate the value of the straight loan.
2. Step 2: Calculate the value of the embedded call.
    a. Calculate the firm's asset volatility at the measurement date using the Merton Model and the average expected enterprise return based on an asset pricing model.
    b. Calculate the term structure of yield spreads at the measurement date.
    c. Calculate forward yield spreads from "b" above.
    d. Simulate an array of future enterprise values using a Monte Carlo framework. We employ a Monte Carlo analysis to generate a 1000 enterprise values per quarter over the remaining life of the loan. These values are based on the analysis undertaken in "a" above.
    e. Calculate the forward credit rating based on the enterprise values generated from step "d" above using Axiom's credit rating platform.
    f. Translate the credit rating to a projected credit spread based on "c" above.
    g. Compare projected market credit spreads to the contractual credit spread. If the projected spread is less than the contractual spread, then the call is activated. If the savings from the spread difference is greater than the call premium plus costs associated with refinancing, then the call is triggered.
    h. At each node where the call is triggered, calculate the coupon savings over the remaining life of the loan. This value is then multiplied by the probability of the event occurring which is based on the Monte Carlo in "d" above. This value is then discounted by the appropriate risk free rate. The sum of these values over the life of the loan is equal to the value of the embedded call.
3. The value of the loan is then equal to the value calculated in step 1 minus the value calculated in step 2h.
4. The value in 3 is then adjusted for lack of liquidity using a put option framework. In this case, the price of illiquidity is measured as the cost of the right to sell the loan at its pre-liquidity value at some point in the future.

Loan Valuation Example

The borrower is a firm that sells software that manages global computer networks of customer firms. The firm provides both maintenance and related value added services including outsourcing solutions that help firms more efficiently integrate in-house databases and local computer networks to the internet.

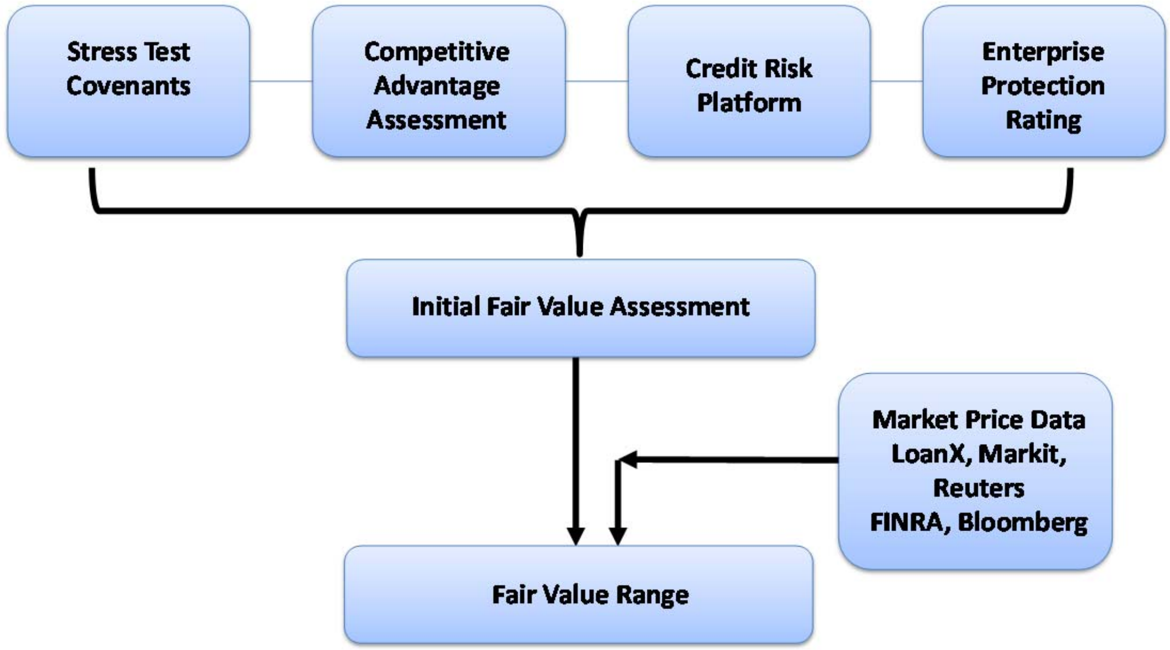
Table 1-1: Loan Description

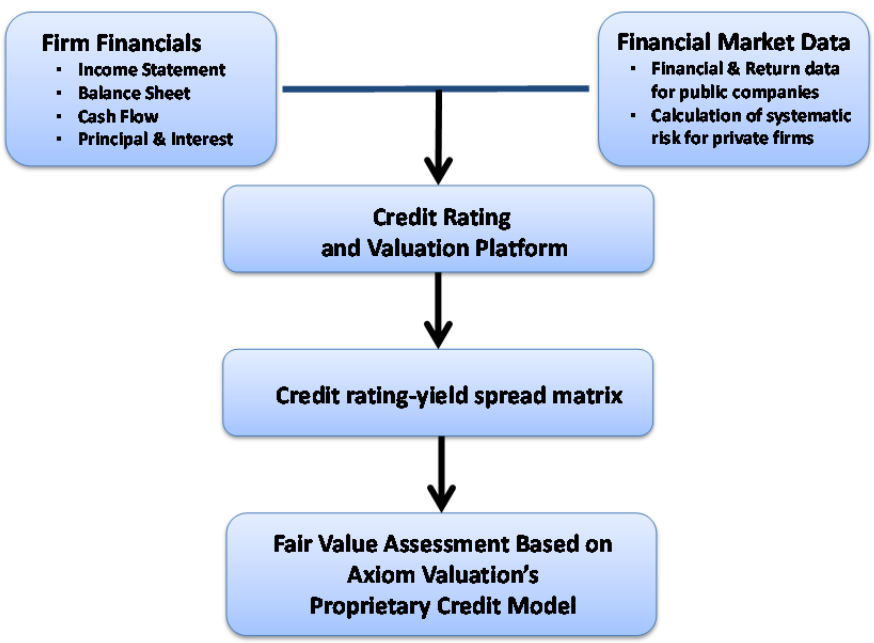
Valuation Date	6/30/2013
Maturity Date	9/30/2017

Enterprise Value	$150,000,000
Debt	$100,000,000
Starting Rating	B-
Coupon	LIBOR + 12%
Coupon Spread on Treasury at Issue Date	9%

Step 1: We first value the straight loan. For this we us Axiom's credit rating platform which is shown below.

Exhibit 1-2: Axiom Credit Rating and Fair Value Platform Overview





The value of the straight loan is calculated as follows:

$$SV = \sum_{t=1}^{n} c_t P_t d_t + P_n d_n$$

Where:

1. SV = value of straight bond
2. c_t = coupon at time t
3. P_t = principal at time t
4. $d_t = 1/(1 + ytm)^t$

Using Axiom's credit platform, the credit rating assigned to the loan was B-. Given the credit rating, the maturity of the loan and credit spreads at the measurement date, the expected *ytm*[1] for this credit is 7.3%. Based on these inputs *SV* = $116,550,000

Step 2a: Since we use a Monte Carlo methodology to value the embedded call, we need to first calculate the appropriate asset return volatility. This is done by first selecting a series of comparable public firms. For each firm we use the Merton framework to simultaneously solve for enterprise value and asset volatility. We then calculate the average asset volatility and use this as input in a Monte Carlo simulation. The Merton results indicate that the average asset volatility is 20%.[2]

Step 2b: The forward rate curve by credit rating is calculated from the credit rating term structure curve at the valuation date. This curve is shown in the top part of Table 1-3.

Table 1-3: Forward Spreads by Rating

Spread Tables

	1-year	2-year	3-year	5-year	7-year	10-year
Ba3/BB-	330	360	383	409	431	454
B1/B+	406	435	459	486	510	535
B2/B	482	511	536	563	589	616
B3/B-	559	586	612	640	668	697
Rf	**0.15**	**0.36**	**0.66**	**1.41**	**1.96**	**2.52**

4-Year Forward Rate

	Current	1-year	2-year	3-year	4-year
BB-	396	429	451	468	482
B+	473	506	530	549	565
B	550	584	609	630	648
B-	626	661	689	711	731

Step 2c: The four year forward curve is shown in the lower part of Table 1-3. These curves are generated based on the "no-arbitrage" condition underlying the expectations theory of the term

[1] *ytm* is the yield to maturity

[2] These calculations are not shown since disclosing the comparable firms may result in disclosing the name of the issuing firm. On the use of the Merton Model see Axiom's paper on valuing a loan in default.

structure.[3] It is assumed that when the security is called, it will be refinanced with a loan of equal maturity to the security it replaced.

Steps 2d- 2f: Project changes in credit risk using a Monte Carlo framework as shown in the table below.

Table 1-4: Forward Credit Ratings

Starting Enterprise Value	$150,000,000

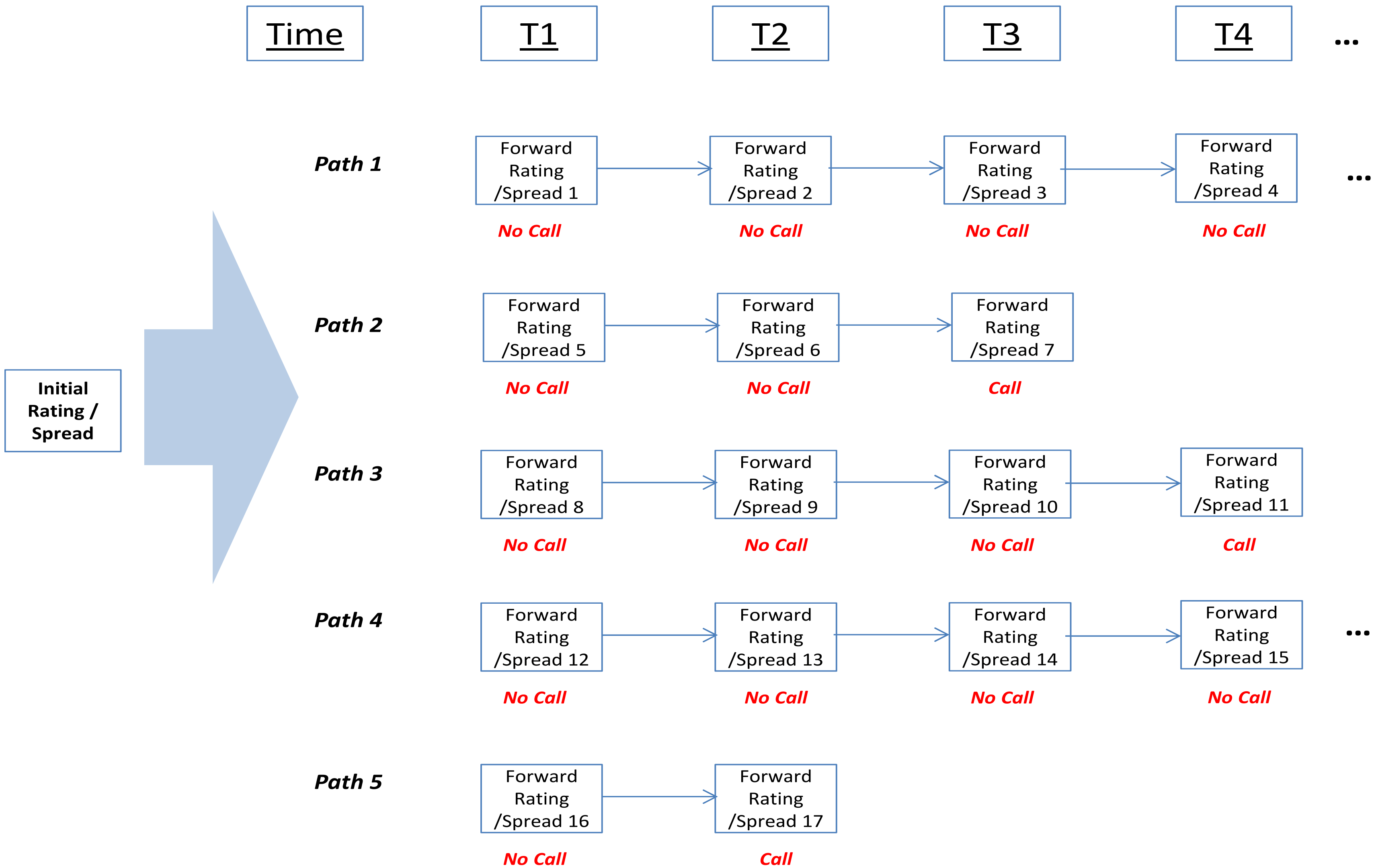
	Return	Enterprise Value	Corresponding D/E Ratio	Corresponding Credit Rating
9/30/2013	4.84%	$157,441,862	1.7409	B-
12/31/2013	0.41%	$158,094,384	1.7213	B-
3/31/2014	6.39%	$168,533,802	1.4591	B
6/30/2014	6.09%	$179,114,187	1.2640	B
9/30/2014	4.88%	$188,064,786	1.1355	B+
12/31/2014	1.58%	$191,058,528	1.0982	B+
3/31/2015	9.18%	$209,424,863	0.9139	BB-
6/30/2015	-2.54%	$204,180,911	0.9599	BB-
9/30/2015	0.05%	$204,287,876	0.9589	BB-
12/31/2015	5.19%	$215,172,016	0.8683	BB-
3/31/2016	6.52%	$229,669,838	0.7712	BB-
6/30/2016	5.67%	$243,072,974	0.6989	BB-
9/30/2016	7.19%	$261,203,564	0.6203	BB-
12/31/2016	4.46%	$273,118,616	0.5776	BB-
3/31/2017	5.45%	$288,423,183	0.5307	BB-
6/30/2017	8.27%	$313,276,576	0.4689	BB-

Based on the calculated asset volatility and the expected return, 1000 enterprise values per quarter over the remaining life of the loan are simulated. Given these values, the projected debt to equity ratio is calculated and then these values become inputs to Axiom's credit rating platform. Based on these inputs, the loan's credit risk is recalculated. Table 1-4 only shows a sampling of outcomes from this exercise. Based on the Monte Carlo framework, debt to equity ratios go down as well as up and in the latter case increases are associated with lower credit ratings which in general do not activate a call.[4] The table below shows a schematic of the simulation.

[3] $(1 + CX)_{t+n})^{t+n} = (1 + CX_n)^n(1 + EX)^t$ where CX is the credit spread in basis points at the measurement date for maturity t and n is the number of years from the measurement date. EX is the expected credit spread for maturity "t" in "n" years.

[4] While not likely, it is possible that a significant narrowing of market credit spreads accompanied by a higher credit risk could trigger a call if the narrowing market credit spread at the higher credit risk is lower than the contractual spread.

Table 1-5: Simulation Demonstration



Steps 2g-2h: The expected forward credit spread by rating is calculated and compared to the contractual coupon spread at each point. If the difference is larger than 4%, we assume that the loan would be called at that point as shown below.

Table 1-6: A Sample of Call Exercise Dates

	Corresponding Credit Rating	4Yr Forward Spread	Coupon Spread	Spread Difference	Result
9/30/2013	B-	626	900	274	No Call
12/31/2013	B-	626	900	274	No Call
3/31/2014	B	550	900	351	No Call
6/30/2014	B	550	900	351	No Call
9/30/2014	B+	506	900	394	No Call
12/31/2014	B+	506	900	394	No Call
3/31/2015	BB-	429	900	471	Call
6/30/2015	BB-	429	900	471	Call
9/30/2015	BB-	451	900	449	Call
12/31/2015	BB-	451	900	449	Call
3/31/2016	BB-	451	900	449	Call
6/30/2016	BB-	451	900	449	Call
9/30/2016	BB-	468	900	432	Call
12/31/2016	BB-	468	900	432	Call
3/31/2017	BB-	468	900	432	Call
6/30/2017	BB-	468	900	432	Call

The four percent threshold reflects transaction costs associated with closing out the initial loan and putting a new loan on the books.

Once the call time is determined, the net cash flow savings is calculated by summing all the remaining coupon payments less a premium, if any. A portion of the "call tree" is shown below.

Table 1-7: Net Cash Flow Calculations

	Result	Remaining Cash Flow Saved	Call Premium	Net Cash Flow Saved	Total Cash Flow Saved at the Call Date
9/30/2013	No Call	N/A	N/A	N/A	N/A
12/31/2013	No Call	N/A	N/A	N/A	N/A
3/31/2014	No Call	N/A	N/A	N/A	N/A
6/30/2014	No Call	N/A	N/A	N/A	N/A
9/30/2014	No Call	N/A	N/A	N/A	N/A
12/31/2014	No Call	N/A	N/A	N/A	N/A
3/31/2015	Call	$1,161,038	$0	$1,161,038	$11,161,641
6/30/2015	Call	$1,173,938	$0	$1,173,938	
9/30/2015	Call	$1,132,538	$0	$1,132,538	
12/31/2015	Call	$1,132,538	$0	$1,132,538	
3/31/2016	Call	$1,120,227	$0	$1,120,227	
6/30/2016	Call	$1,120,227	$0	$1,120,227	
9/30/2016	Call	$1,089,163	$0	$1,089,163	
12/31/2016	Call	$1,089,163	$0	$1,089,163	
3/31/2017	Call	$1,065,485	$0	$1,065,485	
6/30/2017	Call	$1,077,324	$0	$1,077,324	

At each node over the life of the loan the credit market spread at the appropriate credit risk is compared to the contractual spread. If this analysis activates a call, then the savings from refinancing is calculated. Keep in mind that once a call is triggered, the branches emanating from the node disappear. The call value is then equal to:

$$CV = \sum_{t=1}^{n} \sum_{i=1}^{1000} S_{ti} p_i d_t$$

where:

1. S_{ti} = savings from refinancing over the period "t" to "n" at node i.
2. p_i = probability of event at node i.
3. d_t = discount factor using the risk free with t quarters from the valuation date

Table 1-7 below shows this calculation.

Table 1-8: Call Value Calculation

	9/30/2013	12/31/2013	3/31/2014	6/30/2014	9/30/2014	12/31/2014	3/31/2015	...	6/30/2017
Discount Factor	1.00	0.99	0.99	0.99	0.99	0.98	0.98	...	0.96
Cash Flows	$0	$16,352,986	$15,116,372	$14,172,634	$14,138,859	$12,952,021	$11,161,641	...	$1,089,163
Probability	0.00%	0.90%	5.95%	11.37%	0.12%	4.12%	7.84%	...	11.03%

Call Value	$9,287,934

Based on the Monte Carlo, we calculate the probabilities shown above. The value of the loan before liquidity adjustment is then calculated as:

SV-CV = $116,550,000- $9,287,934 = $107,262,066

Since there is no active market for the loan, we adjust its value for lack of liquidity. This is done using an at-the-money put option model where the exercise price is equal to the pre-liquidity adjusted value of the loan, $107,262,066. The value of the put is the cost an owner would be willing to pay to ensure that the loan will sell for its pre-liquidity value at some time in the future. The life of the option is six months and the volatility of the option is calculated as the volatility of a traded ETF high yield bond fund. In the end, we subtract this put option value from the value before liquidity adjustment to obtain the fair value of the loan.

Table 1-9: Liquidity Put Option Calculation

Loan Value Before Liqudity Adjustment=	$107,262,066.00	Interest rate (6 month T-Bill Rate)=	0.10%
Strike Price=	$107,262,066.00	Variance=	0.042
Expiration (in years) =	0.5	Annualized dividend yield= (Interests / Price before liquidity Adjustment)	7.89%
d1 =	-0.19731255		
N(d1) =	0.421791479		
d2 =	-0.341807318		
N(d2) =	0.366247951		
Value of the call =	**$4,227,292.02**	**Value of the put =**	**$8,322,783.62**

The fair value of the loan is calculated as follow:

FV = $107,262,066- $8,322,784 = $98,939,282

Summary and Conclusions

The analysis indicates that issuer's right to call the loan at any time has substantial value and as a result it reduces the value of the loan to the investor. Despite this, it is very possible, although not in this case, that the loan's fair value will exceed par and in some cases by a considerable amount. This would occur if transaction costs associated with refinancing are large, credit spreads are not expected to significantly decline relative to the contractual spread and/or the credit rating is not likely to improve from its assigned value as of the measurement date. In comparison, during stable credit market periods where spreads are narrow and lender competition is significant resulting in lower round trip transaction costs, the value of the call is likely to be low relative to the loan's notional value.