Understanding the Firm Specific Risk Premium*

Stan Feldman† Todd Feldman‡

January 10, 2023

Abstract

We seek to determine whether a firm specific risk premium (FSRP) exists for private firms. We show that private equity investors price firm specific risk as part of establishing the expected rate of return hurdle rate. Our research is based on survey data constructed by the Private Capital Markets Project. We decompose the rate of return into its component parts- market risk premium, size premium, liquidity premium and firm specific risk premium (FSRP). We find that on average PE FSRP varies between zero and six percent. These findings indicate that the cost of capital buildup used in valuing private firms should include a premium for firm specific risk unless facts and circumstances suggest otherwise.

Keywords: Cost of Capital, Firm Specific Risk **JEL codes:** G31, G32

*We would like to thank Olivia Zhao for her work as a research assistant on the paper. We also thank the anonymous referee for clarifying comments and suggestions. We retain sole responsibility for remaining idiosyncrasies and errors.

† Axiom Valuation, Wakefield, MA 01880, USA. *stan@axiomvaluation.com*

‡ Finance Department, San Francisco State University, San Francisco, CA, 94132, USA. *tfeldman@sfsu.edu*

1 Introduction

In this paper we estimate the firm specific risk premium (FSRP) component of a private firm's cost of capital using a novel data set and approach. The motivation for the paper is that the components of the private firm cost of capital, most importantly the firm specific risk (FSR), are not easily measurable. There is no literature we are aware of that has directly addressed how to measure FSRP and its size for private firms. Although there is a significant literature on measuring idiosyncratic risk for public firms and less for private firms, this research generally does not address firm specific risk, which is the focus of our paper. The issue of firm specific risk is important because owners of private businesses as well as their investors, private equity and venture capitalists, are not diversified and hence are subject to nonsystematic risks. This point is underscored by Chen et.al. (2010):

> *Second, due to market incompleteness, the entrepreneur will demand an idiosyncratic risk premium when valuing the firm. We derive an analytical formula for this idiosyncratic risk premium, the key determinants of which are risk aversion, idiosyncratic volatility, and the sensitivity of entrepreneurial value of equity with respect to cash flow. Quantitatively, we show that ignoring the idiosyncratic risk premium can lead to substantial upward bias in firm valuation.*

This point is also made by Fugazy (2017) in reference to private equity investors. Our research builds on the fact that investors in private firms (business owners are investors) have chosen to take on idiosyncratic risk and therefore we should be able to measure the magnitude of the return premiums that make up the returns they expect. We use expected return survey responses of managers of private equity (PE). We then decompose these returns into their systematic and idiosyncratic risk premiums. Our focus is on the latter which we identify as the size risk premium (SRP), the liquidity risk premium (LP), and the firm specific risk premium (FSRP). We combine survey data with other publicly available data sources on public firms to estimate the size and liquidity premiums embedded in the PE

survey data. We then subtract these premiums along with the market risk premium and the risk free rate from the expected PE returns to estimate the residual which is the firm specific risk premium.

We use survey data based on expected returns for two reasons. First, measuring expected premia for different types of risks that informed investors in private firm's face is relevant for those that are valuing private firms since it indicates what informed investors are thinking and the size of the premium for accepting FSR they require. Second, investors in private firms make mistakes and as a result actual returns do not fully reflect all the risks that were initially priced. Although limited partners in PE firms invest in portfolios of private firms, and not in individual firms, the managers of these funds are taking FSR when they invest. Since these portfolios are not diversified, indeed they are often quite specialized by industries[1], one would expect the portfolio alphas to be high enough to account for FSR after accounting for size and liquidity risk.[2]

Our findings indicate that FSRP does exist for private firms and its size is dependent on the amount of equity purchased in the private firm. The average FSRP calculated is around 2.0 percent for PE buyout investments, with a majority ranging between 0 and 6 percent depending on the size of the marketability discount and if the PE investor purchases all of the firm's equity. These results are particularly important for valuation practitioners. The

1 Danielle Fugazy, Special Report on Private Equity: 7 ways PE firms are evolving, Mergers and Acquisitions, July 20, 2017. "As the benefits of sector specialization became proven through data, private equity firms have realigned along industry verticals. The proliferation of sector-focused funds continues, and we don't expect that to change" says Andrea Auerbach, managing director and global head of private investment research at Cambridge Associates.

2 The Capital Asset Pricing Model (CAPM) suggests that idiosyncratic risk should not be priced because it can be eliminated through diversification. Theoretical work by Merton (1987) predicts a positive relation between idiosyncratic risk and stock returns to the extent investors do not fully diversify their portfolios due to market imperfections. The positive relation is also suggested in other early studies such as Lintner (1 965); Levy (1 978); Tinic and West (1986); and Lehmann (1990).

reason being is that there is little literature on whether a FSRP should exist and if so, what its magnitude is given multiple capital structure settings. In this regard, this paper paves an important path for all valuation practitioners. To this end we develop a matrix based on the amount of equity purchased and the percent of debt in the capital structure to guide practitioners in the calculation of FSRP.[3]

2 Background and Related Literature

Are investors in private companies diversified?

Moskowitz and Vissing-Jorgensen (2002) find that investment in private equity to be extremely concentrated. About 75 percent of all private equity is owned by households for whom it constitutes at least half of their total net worth. Heaton and Lucas (2004) document that in the Survey of Small Business Finances, the principal owner of a firm holds on average 81 percent of the firm's equity, and the median owner wholly owns the firm. Other empirical studies include Gentry and Hubbard (2004), Berger and Udell (1998), Cole, Wolken, and Woodburn (1996), and Petersen and Rajan (1994) support this finding.

Although owners of private firms may not be fully diversified, the use of debt may provide a diversification benefit to partially offset the nonsystematic risk the investor in a private firm faces. As Chen (2010) has noted:

> *The diversification benefits of debt are large. Even without any tax benefit of debt, the entrepreneurial firm still issues a significant amount of debt. The diversification benefits also lead to a seemingly counterintuitive prediction: More risk-averse entrepreneurs prefer higher leverage. On the one hand, higher leverage increases the risk of the entrepreneur's equity stake within the firm. On the other hand, higher leverage implies less equity exposure to the entrepreneurial project, making the entrepreneur's overall portfolio (including both his private equity in the firm and his liquid financial wealth) less risky. This overall portfolio composition effect dominates the high*

Professor Ken Ayotte, Berkeley School of Law and Professor Ken Morrison, Columbia Law in their paper "Corporate Disputes in Corporate Bankruptcy", University of Pennsylvania Law Review, Forthcoming Columbia Law and Economics Working Paper UC Berkeley Public Law Research Paper, indicate that the use of FSRP in the discount rate is not supported by finance theory and its use leads to needless disagreement conclusions of value in bankruptcy settings. Our research indicates this conclusion is incorrect, but its size will depend on the firm's capital structure post-bankruptcy.

> *leverage effect within the firm. The more risk-averse the entrepreneur, the stronger the need to reduce his firm risk exposure, therefore the higher the leverage. Second, due to market incompleteness, the entrepreneur will demand an idiosyncratic risk premium when valuing the firm. Quantitatively, we show that ignoring the idiosyncratic risk premium can lead to substantial upward bias in firm valuation.*

Our research indicates that idiosyncratic premium is indeed large, and that more debt in the capital structure results in a larger firm specific risk premium. The view that non-diversified investors require a premium for taking this risk is consistent with the fact that the return on private equity exceeds the return on public equity by about seven percent (16.2 vs 9.6).[4]This is also consistent with alphas earned by limited investors in private equity and VC funds as we show in Table 1. Even so, future developments may lead to greater diversification within the PE space as companies such as NASDAQ are working on establishing a private market exchange and Coller Capital is creating a secondary market in limited PE partnership interests. These new developments should allow investors in PE and VC firm portfolios to buy and sell limited partnership investment interests more easily, but this development does not directly impact managers of these portfolios. They still are pricing non-systematic risk, which is how they make money (typically 20% above a benchmark), which indicates that FSRP may be a component of expected survey returns in excess of any size and/or liquidity premium. Hence, the use of survey data offers a unique way to measure the degree to which FSRP is a component of the discount rates used by investors in private firms when they price their deals.

Does the private firm premium exist and is it large enough to account for FSR?

The empirical results on the size of alphas earned by private equity and venture capital investors before fees may offer some evidence whether these investors price firm specific risk and whether the associated premium shows up in the cash-on-cash returns earned. We

[4] Katya Kartashova, Private Equity Premium Puzzle Revisited, American Economic Review, 2014, 104(10): 3297-3334.

have summarized the empirical work in Table 1. Private equity alphas range from a low of four percent to a high of 22 percent with an average alpha of 10.5 percent and a median of 7.95 percent. VC alphas by comparison range from a high of 32 percent to a low of zero percent with an average and median of 15.96 percent and 8.90 percent respectively. VC alphas are higher on average than private equity alphas as one would expect since the former invest in early-stage firms for the most part while private equity managers invest in firms that are typically cash flow positive and are far more advanced in their business life cycles. Gupta and Van Nieuwerburgh (2019) use a replicating benchmark portfolio of public assets that are comparable to private equity portfolios. They find that on average, PE funds considerably outperform their replicating portfolio benchmark indicating these outsized returns may incorporate firm specific risk premiums.

3 Model: Components of the Expected Private Firm Return

The survey expected return data measures the expected cash-on-cash return investors expect on their investments in a private firm.[5] We approximate this expected return, $R_{irr,}$ using equation (1).

$$R_{irr} = R_f + ERP + SRP + \Delta SRP + LP + FSRP \quad (1)$$

where:

1. R_f = the risk-free rate measured by the 20-year Treasury Bond.
2. ERP = equity market risk premium as measured by Duff and Phelps
3. SRP = size risk premium as measured by Duff and Phelps[6]

[5] The expected cash- on- cash return is the expected internal rate of return. We use expected return and expected IRR interchangeably in this paper.
[6] Duff and Phelps Cost of Capital Yearbook was rebranded under the Kroll name. See https://www.kroll.com/en/cost-of-capital/us-cost-of-capital

4. Δ *SRP* = the incremental size premium for firms smaller than the public firm benchmark
5. *LP* =liquidity premium resulting from the fact that the claim does not trade in an organized liquid market.
6. *FSRP* = firm specific risk premium reflecting the unique characteristics of the private firm

4 Data

The primary data used in the paper is from the Private Capital Markets Project (PCMP), which is an initiative of the Pepperdine Graziadio Business School. The PCMP conducts surveys on private financing to provide insight into the cost of capital for various private suppliers of capital. Table 3 displays a sample of the data. We use the PCMP survey data to create a data set for private equity (PE).[7] PCMP also segments PE firms into two categories, buyout and non-buyout firms. The definition of a buyout (non-buyout) firm is a PE firm that purchases a controlling (non-controlling) interest. The PE data set extends from 2010 to 2021.[8] The data set is unique in that the PE firms report their expectations for the upcoming year as opposed to actual IRRs.

The main variables of interest are the expected returns and proxy variables for the components of the expected returns: size, liquidity, and firm specific risk (FSR). We proxy for size using two variables, the Size of Deal and EBITDA multiples. The size of deal category is broken into seven ranges where the first range includes investments in companies that earn between zero and $999,999 EBITDA per year. The second range includes

[7] The PCMP also surveys VCs. However, data limitations prevented us from developing FSRP estimates for the VC segment.
[8] See https://digitalcommons.pepperdine.edu/gsbm_pcm_pcmr.

investments made in companies that earn between $1 million and $5 million EBITDA per year and so on up until the seventh range, which includes investments in companies earning over $100 million dollars in EBITDA per year. Table 2 shows that as firms become larger the size premium declines. Table 3 and Table 8 show that EBITDA multiple increases with the size of the deal. Therefore, we use EBITDA multiples to proxy for size.

The liquidity proxy we use, *ACtoWF,* is developed as a score using five survey questions to determine whether the number of companies worthy of financing exceeds capital available as displayed in Table 4. For example, in 2019, as shown in Table 4, respondents indicated a reported shortage of capital for those companies with less than $5 million in EBITDA, but a general surplus for companies with $50 million in EBITDA. Therefore, lower EBITDA private firms had a lower score, such as -0.5, which signifies less liquidity whereas higher EBITDA private firms had a higher score, such as 1, signifying more liquidity. Generally, smaller firms with lower EBITDA have a smaller investor following and therefore face greater liquidity risk than their larger counterparts.

An innovation in the paper is that we estimate the PE size and liquidity premium for firms in our sample. We do this by first estimating the initial condition value for size. This reflects the size premium for the smallest public firms, which Kroll, a subsidiary of Duff and Phelps makes available.[9] Table 2 displays size premiums by decile based on market capitalization. The largest companies in the survey sample are in Kroll decile 9, between $235 and $543 million market capitalization. We then calculate the liquidity premium initial condition for the least liquid public firms by appealing to the work of Liu (2004). To calibrate the liquidity initial conditions, we calculate the value of a no growth perpetuity, reduce the value by a

[9] The Duff and Phelps, Cost of Capital Navigator, www.duffandphelps.com. was the source of the data prior to 2018 after which it was rebranded as Kroll, https://www.kroll.com/en/cost-of-capital/us-cost-of-capital

range of marketability discounts reported in the trade literature (Novak (2016), and then recalculate the discount rate that would generate the liquidity discounted value of the no growth perpetuity.[10] The results are shown in Table 6. The values in each cell reflect the increment in the expected return needed to produce the discounted value due to lack of marketability. Hence, if the expected return before adjusting for lack of liquidity were 20 percent, then the expected rate of return would need to rise by 5 percent to 25 percent to produce a value equal to the initial value discounted by a 20 percent marketability discount. Therefore, our model establishes and confirms high thresholds for the size and liquidity premiums of PE investments. Finally, we estimate a regression model to calculate the incremental size and liquidity premiums. The liquidity and size premiums for our sample are therefore the initial benchmarks plus the increment for size and liquidity respectively.

We also estimate a model that explains FSRP. The regressors are the % of total equity purchased, *EquityPurchased,* and equity of WACC, the percentage of equity relative to debt in the capital structure of the firm. This model leverages the work of Chen (2010) which concludes that private firms have more debt in the capital structure than public firms. Chen argues that this is the case because debt provides diversification benefits for owners of private firms because they operate in incomplete capital markets. Our work does not support this hypothesis which is addressed later in this paper.

We use data from the Kroll Cost of Capital book for the equity risk premium and size premiums. Table 5 displays the equity risk premiums by year for the US stock market. Kroll uses a historical equity risk premium approach using stock returns dating back to 1926.

[10] We refer to the discount for lack of liquidity as the marketability discount as customarily used in the trade literature. The premium associated with it is the liquidity premium. As a practical matter, the marketability discount is determined by lack of market liquidity and any restrictions unique to the security being valued. Both characteristics show up in the form of less liquidity and therefore investors require a premium for accepting liquidity risk.

What is interesting is that the long-term market risk premiums are relatively stable, ranging from a low of 6.62% in 2012 to a high of 7.25% in 2021. If we looked at shorter durations, the market risk premiums would show a great deal more variability. But PE investors are relatively longer-term investors and therefore a longer-term market risk premium is more appropriate for our analysis. Pitchbook (2019), a respected data source for the PE community notes that the median holding period for PE investments is five years but can be as high as eleven years based on 2019 data.[11]

Empirical Model

5.1 Private Equity

We calculate FSRP in two different ways to validate the empirical results. The first experiment uses EBITDA multiples and ACtoWF in the same regression to proxy for size and liquidity respectively. In the second experiment we estimate two regressions. In the first we use the EBITDA size groupings to proxy for size. In the second regression we use ACtoWF to proxy for liquidity. We then compare the two FSRP calculations to determine the robustness of our empirical results. In addition, to allow for serial correlation and heteroskedasticity in the residuals from the estimated regressions we rely on Newey and West (1987) robust t-statistics.

5.1.1 Experiment 1

In the first step, we determine the incremental size and liquidity premiums from the "within"

[11] Pitchbook concludes: Overall, we expect holding periods are likely to slowly tick up over the long term, which will necessitate more activity in the GP-led secondaries market, partial sales and more. The proliferation of long-dated funds and nontraditional investors, including SWFs, public pensions and family offices, is changing the PE market. These institutional investors often have a longer time horizon than PE firms or the traditional fund structure and some are willing to hold investments indefinitely. The culmination of partial sales, along with the continued rise of long-dated funds, is likely to affect the exit market for years to come, with exits lagging deals to a greater extent. Furthermore, a rise in nontraditional investors buying assets with an indefinite holding period could cause a rise in deals that never appear as exits.

survey data sample by running experiment 1 / regression 1,

$$E(MedianIRR_{ij,t}) = \beta_0 + \beta_1 ACtoWF_{ij,t} + \beta_2 EBIT\ DAmultiple_{ij,t} + \epsilon_{ij,t}, \quad (1)$$

where $E(MedianIRR_{ij,t})$ is the median expected return within EBITDA range i and PE group j (buyout or non-buyout). The liquidity variable $ACtoWF_{ij,t}$ stands for available capital to worthy of financing for EBITDA group i and PE group j. The variable EBITDA Multiple proxies for size. Lastly, ϵ_t is the regression error term.

In the second step, we use the coefficients and constant from equation (1) to determine the incremental premiums from within the sample and then add these differences to the initial conditions- base values- for size and liquidity respectively. Equation 2 shows this set up.

$$\begin{aligned} &Size\ Liquidity1_{ij,t} = \\ &\quad +\beta_1(ACtoWF_{ij,t} - ActoWF7_{j,t}) + \beta_2(EBITDAmultiple_{ij,t} - EBITDAmultiple7_{j,t}) \\ &\quad +size\ premium7_{j,t} + liquidity7_{j,t}, \end{aligned} \quad (2)$$

Where $\beta 1(ActoWFij,t\text{-}ActoWF7j,t)$ and $\beta 2(EBITDAmultipleij,t\text{-}EBITDAmultiple7j,t)$ equal the incremental liquidity and size premiums found within the sample. The variables *size premium7j,t* and *liquidity7j,t* reflect initial conditions (base levels) where the former is the size premium associated with the smallest Kroll decile. The liquidity initial condition is based on Table 6, which shows the incremental return associated with various marketability discounts and their associated costs of capital.

Since our focus is on private firms, the base liquidity premium should be the average premium associated with the most illiquid segment of the public markets. Liu (2004) estimates a two variable asset pricing model which includes a market premium (CAPM) and a liquidity variable. He then measures the incremental return between the least liquid and most liquid stocks. He shows that this model is superior to the three factor Fama French

model (1991). We use Liu's results to set the initial conditions for our liquidity variable. The reason is that the firms that private equity investors typically invest in are cash flow positive and distributions are often made to investors as a result. Liu shows that firms that have this characteristic have far lower liquidity premiums than firms that do not distribute cash. Liu reports the annual incremental premium ranges between 2.71 % and 5.78% with an average of 4.25%.[12] Using Table 6, this centers on a combination of cost of capital between 20% and 25% and marketability discounts between 20% and 25%. We use these characteristics to set the liquidity premium initial condition value for PE investments.

In step three, we calculate the firm specific risk premium, $FSRP1$, by subtracting the risk free rate, rf, equity risk premium, ERP, and the estimated size and liquidity premiums,

$$FSRP1_{ij,t} = E(MedianIRR_{ij,t}) - rf_t - ERP_t\text{-}SizeLiquidity1_{ij,t}, \quad (3)$$

where the risk-free rate is calculated as the 20-year Treasury at the beginning of the year and ERP_t is calculated using the historical equity risk premiums in Table 5.

5.1.2 Experiment 2

In experiment 2 we run two separate regressions to estimate the size and liquidity premiums. First, we run experiment 2 - regression 1a to estimate the size premium using the expected median IRR as the dependent variable and indicator variables for the various size groups where the constant is the coefficient for the smallest size group as indicated in the equation below.

$$E(MedianIRR_{ij,t}) =$$

$$\beta_0 + \beta_1 Size2_{j,t} + \beta_2 Size3_{j,t} + \beta_3 Size4_{j,t} + \beta_4 Size5_{j,t} + \beta_5 Size6_{j,t} + \beta_6 Size7_{j,t} + \epsilon_{ij,t}, \quad (4)$$

[12] Liu, Table X, panels A and C.

Each indicator variable equals one if the EBITDA is within size group i and zero otherwise. As noted, the constant equates to the median expected IRR associated with companies in the smallest EBITDA range, companies earning under $1 million in EBITDA. We use the regression results from experiment 2 - regression 1a to calculate the size premium defined as *Size*2,

$$Size2_{ij,t} =$$

$$\beta_6 * size1_{j,t} + (\beta_1 - \beta_6) * size2_{j,t} + (\beta_2 - \beta_6) * size3_{j,t} + +(\beta_3 - \beta_6) * size4_{j,t}$$

$$+(\beta_4 - \beta_6) * size5_{j,t} + (\beta_5 - \beta_6) * size6_{j,t} + size\ premium7_{j,t}, \quad (5)$$

where *Size*2 is the incremental size premium between the largest EBITDA range investments and the smaller EBITDA range investments plus $premium7_{j,t,}$ the size premium's initial condition noted above. Next, we run experiment 2 - regression 1b using the expected median IRR as the dependent variable and available capital to worthy of financing as the independent variable,

$$E(MedianIRR_{ij,t}) = \beta_0 + \beta_1 ACtoWF_{ij,t} + \epsilon_{ij,t}, \quad (6)$$

where *ACtoWFij,t* proxies for the liquidity premium. Liquidity2 equals the incremental liquidity premium between the surveyed investments plus the initial condition liquidity premium, *liquidity*7*j,t*,

$$\text{Liquidity2}_{ij,t} = \beta 1(\text{ACtoWF}_{ij,t} - \text{ACtoWF7}_{j,t}) + \text{liquidity7}_{j,t}. \quad (7)$$

Next, we calculate the FSRP in experiment 2, *FSRP*2, as

$$FSRP2_{ij,t} = E(MedianIRR_{ij,t}) - rf_t - ERP_t - Size2_{ij,t} - Liquidity2_{ij,.} \quad (8)$$

5.1.3 FSRP

Chen et.al. (2010) develops a model for the valuation of the private firm which shows that

nonsystematic risk is related to the firm's capital structure since debt provides diversification benefits for the entrepreneurs. Following Chen we develop a model, equation 10, where our estimate of *FSRP1* is regressed against the percent of equity purchased, *EquityPurchased*, and the proportion of equity versus debt in the private firm's capital structure.

$$FSRP1 = \beta_0 + \beta_1 \%EquityPurchased_{ij,t} + \beta_2 \%Equityof\ WACC_{ij,t} + \epsilon_{ij,t}, \qquad (10)$$

where the $\%EquityPurchased_{ij,t}$ equals the median percent of equity purchased for size category i and PE group j. It is a proxy for non-systematic risk. The variable $\%EquityofWACC_{ij,t}$ equals the percent of equity in the capital structure of a private firm of size category i and PE group j. Hypothesis one is that the more equity purchased the greater the FSRP as PE firms are less diversified and take on more non- systematic risk. Hypothesis two is that the greater the financial leverage the greater the FSRP as the credit risk of the firm increases and its equity cushion is reduced. This hypothesis indicates that the credit risk impact is negative and exceeds the positive diversification impact of more debt (Chen (2010)) in the capital structure.

6 Results

Table 7 displays summary statistics for the PE data. There are 121 expected IRR observations. The mean expected IRR in the survey is 25 percent with a standard deviation of 5 percent. Table 8 displays summary statistics of the main variables used in the analysis by EBITDA group. Results from Table 8 indicate as deal size increases, the EBITDA multiple and available capital relative to worthy of financing increases as the expected IRR declines. The distribution of the surveyed IRRs indicates that most of the IRRs are within the range of 20 to 30 percent. Table 9 displays the correlation coefficients between the main variables used in the analysis. As expected, size (EBITDA Multiple) and liquidity (worthy

of financing relative to the number of firms) are negatively correlated to IRR. The table also shows that the greater liquidity the higher the multiple where the latter proxies for firm size.

Table 10 displays the results of PE experiment 1 / regression 1. Both the size and liquidity variables are statistically significant, and their respective coefficient signs are negative as expected. As size increases and liquidity improves the expected IRR declines.[13] The size and liquidity factors explain 41.35 percent of the variation in the expected IRR. We use the coefficients in Table 10 to calculate *FSRP*1 as displayed in Figure 1, left. The majority of *FSRP*1s are between 0 and 6 percent with an average of 2.05 percent, a median of 1.89 percent, and a standard deviation of 3.88 percent. Figure 1 indicates that 85 percent of the *FSRP*1 observations are greater than zero and, therefore, support the conclusion that a positive expected firm specific risk premium is a component of the expected PE IRRs.

Table 11 displays the estimated coefficients from PE experiment 2 / regression 1a. The results indicate that the size premium difference between firms earning $100 million EBITDA plus and firms earning less than $1 million EBITDA is approximately 9 percent (-0.088). The size premium declines as EBITDA increases. Therefore, the size premium for the smallest sized investments in the survey is 11.7 percent, a base of 2.7 percent from Table 2, decile 9 (companies earning more than 100 million EBTIDA) plus 9 percent based on the regression results shown in Table 10.

Table 12 displays the liquidity coefficients from Experiment 2 / regression 1b. The highest score in the data sample for ACtoWF is 1.8 which when entered into the regression yields an expected IRR of 19.00 percent. The lowest ACtoWF score in the sample is -0.7, which

[13] The EBITDA multiple and ACtoWF, the liquidity variable, are significantly positively correlated as shown in Table 9. This implies that the standard errors would be biased upward which increases the likelihood that one or both variables would be statistically insignificant. The fact that this is not the case indicates the variables have a significant independent effect on the expected rate of return.

yields an expected return of 30.0 percent, resulting in a 11 percent liquidity premium between the lowest and highest liquid firm in the sample. The liquidity initial condition was set at 2.2 percent (Table 6, IRR 20 percent, marketability discount of 10 percent). In experiment 2 size and liquidity were modeled separately while in experiment 1 they were modeled jointly. Based on the correlation table we know that size and liquidity are positively correlated so the liquidity variable in experiment 2 is picking both liquidity and size. Therefore, by using a lower initial condition value, we are adjusting for this upward bias. Hence, the lowest liquid investments should command a 13.2 percent, (30.0% - 19.00% + 2.2%), liquidity premium over the highest liquid investments.

Figure 1, right, shows the FSRP2 distribution. The mean of *FSRP*2 is 1.61 percent where a one standard deviation range is between -2.96 percent and 6.19 percent. We do get a wider range of firm specific risk premiums than we do calculating FSRP1.

The distributions shown in Figure 1 appear to indicate that FSRP1 and FSRP2 come from different populations. To test this hypothesis, we performed the Kolmogorov-Smirnov test. The results shown in Table 13 indicate we cannot reject the null hypothesis- the distributions are different. This lends credibility to adjusting the liquidity initial condition in experiment 2 and our conclusion that FSRP is best approximated by FSRP1.

Table 14 displays the estimated coefficients from running experiment 1 / regression 2: *FSRP*1 against equity purchased and equity of WACC. Both coefficients are statistically significant at the 5 percent level and the signs are as expected. Table 15 is based on the Table 14 regression results. For a given capital structure, *FSRP*1 increases as the percent of equity purchased increases. This is related to taking on more non-systematic risk. Alternatively, for a given equity purchase, *FSRP*1 declines as the debt-to-equity ratio decreases. This result

indicates that the diversification benefit of debt does not reduce firm specific risk as indicated by Chen. Rather, our result is consistent with the view that the credit risk of the firm is less the more equity in the capital structure and this effect more than compensates for the positive diversification effect of higher debt levels. For example, when a PE investor purchases 100 percent of the firm resulting in a capitalization structure of 70 percent debt and 30 percent equity, the firm specific risk premium required is 4.18 percent. By comparison FSRP would increase to 5.03% if the capital structure were 80 percent debt and 20 percent equity.

We also run regression 2a using *FSRP*2 as the dependent variable.[14] Regression results are similar to those in Table 15, for a given capital structure, *FSRP*2 increases (decreases) as the percent of equity purchased increases (decreases).

7 Conclusion

The goal of the paper is to determine the existence and size of the firm specific risk premium for private firms, and to the extent the premium exists, identify the factors that determine it. This research is particularly important for the determination of the private firm's cost of capital and in turn the value of the private firm.

The private firm's cost of capital is constructed by adding a series of expected return premiums reflecting identifiable specific risks to the risk-free rate. Therefore, we should be able to extract these premiums, including the premium for firm specific risk, from returns expected by knowledgeable investors when they invest in private firms. To do so, we use expected return data for private investors based on survey data developed by Private Capital Markets Project (PCMP). To our knowledge, we are the first to use this data set to estimate

[14] Results will be provided on request.

the expected firm specific risk premium that PE investors in private firms require.

We use a two-step methodology. First, we determine incremental size and liquidity premiums from within the sample for the private firms. Second, we add those premiums to the premiums calculated from public company data which include premiums for equity risk, size risk and liquidity risk for the largest private firms. Together we subtract these premiums (incremental plus outside sample premiums) from returns that PE investors in private firms expect to determine whether a residual premium remains. We term this residual the firm specific risk premium (FSRP). We find that on average FSRP is 2 percent for PE investments. The size of FSRP is a function of the percent of equity purchased and equity in the capital structure. In cases where 100 percent of the equity is purchased, and the firm has a capital structure of 30 percent equity and 70 percent debt, FSRP will be greater than 5 percent. To the extent that a private firm's cost of capital should include FSRP, and it does not, means that the calculated cost of capital is too low. Since all methods used to value a private firm-income, guideline, and transaction- are dependent on the target firm's cost of capital, it follows that a cost of capital that is too low means that the conclusion of value will necessarily be too high. Hence, the valuation implications of the existence of FSRP for private firms and its magnitude are significant.

8 References

Ang, A., Chen, B., Goetzmann, W., & Phalippou, L. 2013. Estimating Private Equity Returns From Limited Partner Cash Flows. Working paper, Columbia University.

Axelson, U., Sorensen, M., & Stromberg, P. 2013. The Alpha and Beta of Buyout Deals. Working Paper, Columbia Business School.

Baele, Lieven, Bekaert, Geert, & Inghelbrecht. 2010. The Determinants of Stock and Bond Comovements, The Review of Financial Studies, Vol. 23: 2374-2428.

Berger, A. N. & G. F. Udell. 1998. The Economics of Small Business Finance: The Roles of Private Equity and Debt Markets in the Financial Growth Cycle. Journal of Banking and Finance.

Buchner, A. 2014. The Alpha and Beta of Private Equity Investments, Working Paper, University of Passau, Germany.

Chen Hui, Miao J. & Wang N., 2010. Entrepreneurial Finance and Nondiversifiable Risk, Review of Economic Studies, Vol 23, No 12, pp.4348-4388.

Cochrane, J. 2005. The Risk and Return on Venture Capital, Journal of Financial Economics, 75, 3-52.

Cole, R. A., J. D. Wolken, and R. L. Woodburn. 1996. Bank and Nonbank Competition for Small Business Credit: Evidence from the 1987 and 1993 National Surveys of Small Business Finances. Federal Reserve Bulletin November: 983-95.

Ewens, M. (2009). A New Model of Venture Capital Risk and Return. Working Paper, University of California San Diego.

Franzoni, F., Nowak, E., & Phalippou, L. (2012). Private Equity Performance and Liquidity Risk. Journal of Finance 67(6), 2341-2373.

Fugazy, D., (2017). Special Report on Private Equity: 7 ways PE Firms are Evolving. Mergers and Acquisitions.

Gentry, W. M., and R. G. Hubbard. 2004. Entrepreneurship and Household Saving. Advances in Economic Analysis and Policy: Vol. 4: No. 1, Article.

Gupta, A., and Nieuwerburgh, Stijn Van. Valuing Private Equity Investments Strip by Strip, CBS, NBER, and CEPR.

Heaton, J. C, and D. J. Lucas. 2004. Capital Structure: Hurdle Rates, and Portfolio Choice Interactions in an Entrepreneurial Firm. Working Paper, Northwestern University.

Korteweg, A., & Sorensen, M. 2010. Risk and Return Characteristics of Venture Capital Backed Entrepreneurial Companies. 2010. Review of Financial Studies Vol. 23 (10), 3738-3772.

Lehmann, B. 1990a. Residual Risk Revisited. Journal of Econometrics Vol. 45: 71-97.

Levy, Haim. 1978. Equilibrium in an Imperfect Market: A Constraint on the Number of Securities in the Portfolio. The American Economic Review, Vol. 68 (4): 643-658.

Linter J. 1965. Security Prices, Risk, and Maximal Gains From Diversification, The Journal of Finance, Vol. 20, No. 4: pp. 587-615.

Merton, Robert. 1987. A Simple Model of Capital Market Equilibrium with Incomplete Information. Journal of Finance, Vol. 42 (3): 483-510.

Moskowitz, T., & Vissing-Jrgensen, A. (2002). The Returns to Entrepreneurial Investment: A Private Equity Premium Puzzle? The American Economic Review, Vol. 92 (4), 745-778.

Newey, W. and Kenneth West. 1987. A simple positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix. Econometrica, Vol. 55: 703-708.

Novak, N. 2016. Measuring the Discount for Lack of Marketability for Noncontrolling, Nonmarketable Ownership Interests. Insights, Winter, 37 to 51.

Petersen, M. A., and R. G. Rajan. 1994. The Benefits of Lending Relationships: Evidence from Small Business Data. Journal of Finance Vol. 49, 3-37.

The Kroll (Duff & Phelps) 2022 Valuation Handbook Cost of Capital. Kroll.

Tinic, S., & West, R. 1986. Risk, Return, and Equilibrium: A Revisit, Journal of Political Economy, Vol. 94, No. 1, 126-147

9 Figures and Tables

Table 1: Estimated Alphas From the Literature

Author	Time Period	Data Set	VC	PE
Axelson[1]	1994-2007			8.3%- 8.6% (Beta=2.2-2.4)
Franzoni[2]	1975-2007	CPER, TVE		9.3% (Beta = .95)
Buchner[3]	1980-2009	CPER, TVE	8.9% (Beta = 2.6)	7% (Beta =2.2)
Cochrane[4]	1987-2000	Venture One	32% (Beta = 1.9)	22% (Beta = .5)
Ang[5]	1993-2008	Preqin	5%*	5%*
Korteweg,Sorensen[6]		VentureXpert	30%	
Ewens[7]	1980-2007	VE, Preqin	0%(Beta =1.24)	4%(Beta = .72)
Average			15.96%	10.48%
Median			8.90%	7.95%

* = net of fees. 1. Axelson, U., M. Sorensen, and P. Stromberg (2013). The alpha and beta of buyout deals. Working paper, Columbia Business School, 2. Franzoni, F., E. Nowak, and L. Phalippou (2012). Private equity performance and liquidity risk. Journal of Finance 67 (6), 23412373. 3. Buchner, Axel (2014). The Alpha and Beta of Private Equity Investments. Working paper, University of Passau. 4.Cochrane, John (2005). The risk and return of venture capital. Journal of Financial Economics 75, 3-52. 5. Ang, A., B. Chen, W. N. Goetzmann, and L. Phalippou (2013). Estimating private equity returns from limited partner cash flows. Working paper, Columbia University 6. Korteweg, A. and M. Sorensen (2010). Risk and return characteristics of venture capital backed entrepreneurial companies. Review of Financial Studies 23 (10), 37383772. 7. Ewens, M. (2009). A new model of venture capital risk and return. Working paper, University of California San Diego.

Table 2: Size Premiums by Decile

Decile	MC-	MC+	2022	2021	2020	2019	2018	2017	2016	2015	2014	2013	2012	2011	2010
10b	$1.06	$19.04	7.89%	8.12%	7.89%	8.25%	8.41%	8.64%	8.76%	8.94%	8.99%	9.74%	9.81%	10.06%	10.01%
10z	$1.06	$12.77	11.17%	11.29%	11.17%	11.14%	11.38%	11.63%	11.79%	11.98%	12.12%	11.65%	11.77%	12.06%	12.06%
10y	$12.79	$19.04	6.34%	6.60%	6.34%	6.85%	6.98%	7.21%	7.32%	7.51%	7.55%	8.90%	8.93%	9.15%	9.05%
10a	$19.05	$28.90	3.31%	3.49%	3.31%	3.71%	3.86%	4.09%	4.04%	4.22%	4.40%	4.23%	4.34%	4.55%	4.45%
10x	$19.05	$25.15	4.54%	4.65%	4.54%	4.68%	5.04%	5.33%	5.30%	5.54%	5.67%	4.66%	4.75%	4.96%	4.91%
10w	$25.17	$28.90	2.34%	2.60%	2.34%	2.89%	2.91%	3.10%	3.04%	3.18%	3.52%	3.66%	3.80%	3.99%	3.85%
10	$1.06	$28.90	4.80%	5.01%	4.80%	5.22%	5.37%	5.59%	5.60%	5.78%	5.99%	6.03%	6.10%	6.36%	6.28%
9	$29.00	$62.78	2.10%	2.29%	2.10%	2.46%	2.50%	2.68%	2.54%	2.69%	2.81%	2.70%	2.80%	2.94%	2.85%
8	$62.91	$130.60	1.21%	1.46%	1.21%	1.80%	1.88%	2.08%	2.04%	2.15%	2.36%	2.46%	2.51%	2.65%	2.49%
7	$130.64	$216.45	1.34%	1.54%	1.34%	1.58%	1.58%	1.72%	1.62%	1.71%	1.94%	1.73%	1.77%	1.88%	1.73%
6	$217.03	$327.66	1.18%	1.37%	1.18%	1.50%	1.60%	1.66%	1.63%	1.74%	1.86%	1.72%	1.75%	1.82%	1.73%
5	$328.10	$500.37	0.89%	1.09%	0.89%	1.28%	1.38%	1.51%	1.49%	1.60%	1.75%	1.70%	1.74%	1.81%	1.69%
4	$501.99	$821.26	0.54%	0.75%	0.54%	0.85%	0.90%	0.98%	0.99%	1.06%	1.16%	1.14%	1.17%	1.20%	1.15%
3	$821.64	$1,673.84	0.55%	0.71%	0.55%	0.81%	0.87%	0.89%	0.86%	0.91%	0.86%	0.92%	0.94%	1.01%	0.85%
2	$1,675.94	$3,609.92	0.43%	0.49%	0.43%	0.52%	0.56%	0.61%	0.57%	0.63%	0.75%	0.76%	0.78%	0.81%	0.74%
1	$3,616.06	$232,439.02	-0.22%	-0.22%	-0.22%	-0.30%	-0.31%	-0.35%	-0.36%	-0.36%	-0.37%	-0.37%	-0.38%	-0.38%	-0.37%

Note.-Table 2 is a table reported by Kroll 2022. Kroll calculated the size premium by market capitalization. MC is market capitalization. MC - is the lower end of the rage and + is the upper range for the average market capitalization between years 2010 and 2022.

Table 3: PE Sample from PCMR 2019 Report - Buyout Transactions

	EBITDA						
Variable	**$0-$999K**	**$1-$4.99M**	**$5-$9.99M**	**$10-$24.99M**	**$25-$49.99M**	**$50-$99.99M**	**$100M+**
Number of investments	8	25	10	7	8	2	5
Average size of investment	1.5	5	15	n/a	55	75	100+
Expected time to exit	> 7	5	5	n/a	5	5	7
Equity as % of WACC	45%	45%	45%	60%	45%	35%	35%
% of equity purchased	85%	75%	75%	70%	35%	35%	35%
Average EBITDA multiple	3	5	5	5	7	7.5	8
Average revenue multiple	1.3	1.5	2	2	4.5	5	5
Median expected returns	37%	28%	28%	25%	25%	22.50%	21%

Note. Table 3 displays a sample from the 2019 Private Capital Market Report in the Private Equity Survey Section; Table 35. General Characteristics Buyout Transactions (medians). The variables are grouped by EBITDA. The expected time to exit, equity as a % of WACC, and % of total equity purchased are median numbers. The median total expected returns are gross cash on cash pre-tax IRRs. Respondents reported on where they plan to invest over the next 12 months. The results reflect investment throughout the U.S.

Table 4: The Balance of Available Capital with Quality Companies by EBITDA Size

	Companies worthy of financing GREATLY exceed capital available (capital shortage)	Companies worthy of financing exceed capital available	General balance between companies worthy of financing and capital available	Capital available exceeds companies worthy of financing	Capital available GREATLY exceeds companies worthy of financing (capital surplus)	**Score**
$0K - $999K EBITDA	23%	19%	26%	23%	10%	**-0.2**
$1M - $4.99M EBITDA	12%	6%	38%	32%	12%	**0.3**
$5M - $9.99M EBITDA	4%	12%	23%	35%	27%	**0.7**
$10M - $14.99M EBITDA	4%	0%	30%	22%	43%	**1.0**
$15M - $24.99M EBITDA	4%	0%	22%	30%	43%	**1.1**
$25M - $49.99M EBITDA	4%	4%	13%	39%	39%	**1.0**
$50M - $99.99M EBITDA	0%	9%	14%	36%	41%	**1.1**
$100M+ EBITDA	10%	0%	19%	24%	48%	**1.1**

Note. Table 4 displays a sample from the 2019 Private Capital Market Report in the Private Equity Survey Section in regard to the available capital with quality companies' survey.

Table 5: US Equity Risk Premium

Year	Historical Long-term (1926 - Present)
2010	6.67%
2011	6.72%
2012	6.62%
2013	6.70%
2014	6.96%
2015	7.00%
2016	6.90%
2017	6.94%
2018	7.07%
2019	6.91%
2020	7.15%
2021	7.25%

Note.-Table 5 is a table reported by Kroll 2022. Kroll calculates the equity risk premium for the US stock market beginning in 1926 up until the year of interest.

Table 6: Liquidity Discount Table

Cost of Capital	Liquidity Discount						
CC	10%	15%	20%	25%	30%	35%	40%
10%	1.1%	1.7%	2.5%	3.3%	4.3%	5.4%	6.7%
15%	1.7%	2.6%	3.7%	5.0%	6.4%	8.1%	10.0%
20%	2.2%	3.5%	5.0%	6.7%	8.6%	10.8%	13.3%
25%	2.8%	4.4%	6.2%	8.3%	10.7%	13.5%	16.7%

Note.-Table 6 are calculated liquidity premiums based on expected IRRs and liquidity discounts.

Table 7: PE Summary Statistics

Variable	Obs	Mean	Stdev	Min	Max
Time to Exit	121	4.45	0.99	1	7
Equity of WACC	121	0.44	0.18	0.05	0.95
Equity Purchased	121	0.57	0.27	0.05	1
EBITDAMultiple	121	6.23	1.61	3	10
Median IRR	121	0.25	0.05	0.12	0.5
ACtoWF	121	0.57	0.61	-0.7	1.8

Note.-Table 7 is a summary table of the main variables used in the PE data analysis. The PE data sample is annual from 2010 to 2021. Time to exit is the time to exit an investment. Equity of WACC is the percent of equity in the WACC for each PE firm. Equity purchased is the amount of equity purchased in the buyout or non-buyout investment. ACtoW is proxy for liquidity.

Table 8: PE Summary by EBITDA Group

Variable	EquityPurchased	EquityofWacc	EBITDAMult	Expected IRR	ACtoWF
0-999k EBITDA	0.62	0.52	4.48	0.30	-0.40
1-4.99M EBITDA	0.57	0.42	5.18	0.27	0.01
5-9.99M EBITDA	0.62	0.45	5.78	0.25	0.45
10-24.99M EBITDA	0.52	0.43	6.49	0.23	0.75
24-49.99M EBITDA	0.58	0.38	7.55	0.22	0.89
50-99.9M EBTIDA	0.51	0.40	7.83	0.21	1.06
100M EBITDA	0.60	0.37	7.72	0.21	0.96

Note.-Table 8 is a summary table of the main variables used in the PE data set by deal size. Deal size is broken up into seven ranges. The data sample is annual from 2010 to 2021.

Table 9: Correlation of Variables

	Median IRR	Average EBITDA Multiple	Capital / Worth of Financing	Equity as a % of WACC	Equity Purchased	Time to Exit
Median IRR	1					
Average EBITDA Multiple	-0.5111	1				
Capital / Worth of Financing	-0.5573	0.6818	1			
Equity as a % of WACC	0.1108	-0.142	-0.2209	1		
Equity Purchased	0.3442	-0.0349	-0.0567	0.2921	1	
Time to Exit	0.2015	-0.2679	-0.0707	0.1137	0.217	1

Note.-Table 9 is a correlation matrix of the main variables used in the PE data set. The data sample is annual from 2010 to 2021.

Table 10: PE Experiment 1 - Regression 1: Median IRR vs. EBITDA Multiple & Available Capital to Worthy of Financing

Variable	Coef.	Std. Err.	Pvalue
EBITDA Multiple	-0.008**	(0.003)	0.029
ACtoWF	-0.036**	(0.009)	0.000
Constant	0.311**	(0.021)	0.000

Note.-Table 10 displays the regression results from regression 1 using PE data. *EBITDA Multiple* is the average EBITDA multiple across surveyed PE investments. It is a proxy for size. *ACtoWF* is the available capital to companies' worthy of financing. It is a proxy for liquidity. The regression is run using Newey standard errors. The R-squared is 41.35%. *significant at 5%; **significant at 1%.

$$MedianIRR_{ij,t} = \beta_0 + \beta_1 ACtoWF_{ij,t} + \beta_2 EBITDAmultiple_{ij,t} + \epsilon_{ij,t.}$$

Table 11: PE Experiment 2 - Regression 1a: Median IRR vs. Size of Deal

Variable	Coef.	Std. Err.	Pvalue
EBITDA 5Mil	-0.031†	(0.016)	0.061
EBITDA_10Mil	-0.051**	(0.016)	0.002
EBITDA_25Mil	-0.069**	(0.017)	0.000
EBITDA_50Mil	-0.077**	(0.016)	0.000
EBITDA_100Mil	-0.091**	(0.018)	0.000
EBITDA_100Mil+	-0.088**	(0.018)	0.000
Constant	0.2990	(0.014)	0.000

Note.-Table 11 displays regression results from running median IRR against the six of the seven size of deal ranges. The constant term is the median IRR for investments in the smallest EBITDA range. The regression is run using Newey standard errors. The R-squared is 41.34%. †significant at 10%, *significant at 5%; **significant at 1%.

$$E(MedianIRR_{ij,t}) = \beta_0 + \beta_1 Size2_{j,t} + \beta_2 Size3_{j,t} + \beta_3 Size4_{j,t} + \beta_4 Size5_{j,t} + \beta_5 Size6_{j,t} + \beta_6 Size7_{j,t} + \epsilon_{ij,t}$$

Table 12: PE Experiment 2 - Regression 1b: Median IRR vs. Available Capital to Companies Worthy of Financing

Variable	Coef.	Std. Err.	Pvalue
ACtoWF	-0.044**	(0.006)	0.000
Constant	0.269**	(0.005)	0.000

Note.-Table 12 displays the second regression in experiment two. Running the median IRR against a liquidity proxy, ACtoWF. The regression is run using Newey standard errors. R-squared is 36.61%. *significant at 5%; **significant at 1%.

$$MedianIRR_{ij,t} = \beta_0 + \beta_1 ACtoWF_{ij,t} + \epsilon_{ij,t.}$$

Figure 1: Distribution of FSRP1 & FSRP2

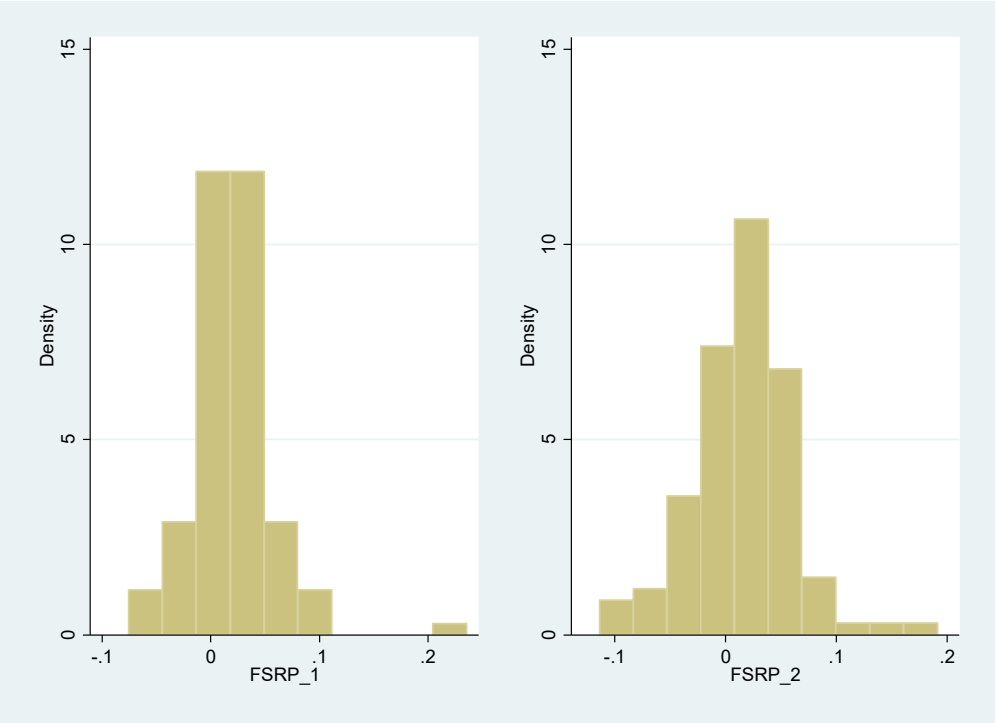

Figure 1 displays the distribution of estimated *FSRP* 1 and *FSRP* 2 for private equity investments from experiment 1, sampled from 2010 to 2021.

Figure 2: FSRP 1 and 2

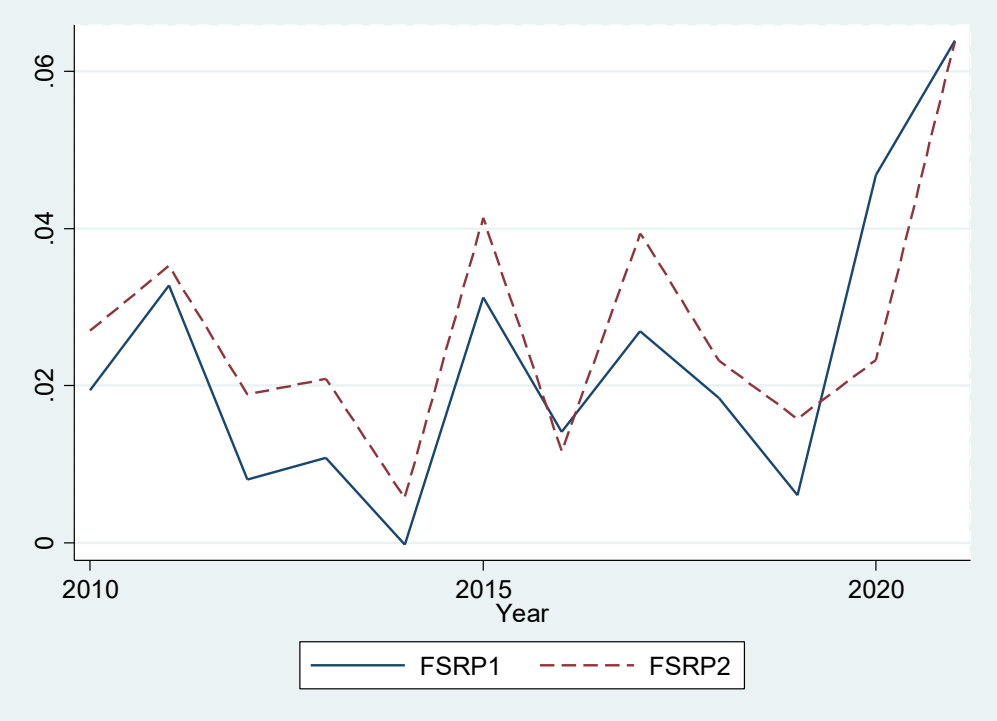

Figure 2 displays the average FSRP for experiments 1 and 2. *FSRP* is averaged across the different size groups by year. Data is from 2010 to 2021.

Table 13: Kolmogorov-Smirnov Test: FSRP1 vs. FSRP2

Group	D	Pvalue
FSRP1	0.0776	0.484
FSRP2	-0.1012	0.291
Combined K-S:	0.1012	0.568

Note.-Table 13 displays Kolmogorov-Smirnov test.

Table 14: PE Regression 2: *FSRP*1 vs. Equity Purchased and Equity % of WACC

Variable	Coef.	Std. Err.	Pvalue
EquityPurchased	0.042**	(0.013)	0.001
EquityofWACC	-0.040*	(0.019)	0.036
Constant	0.019†	(0.010)	0.056

Note.-Table 14 displays the regression results from experiment 1, regression2. *FSRP*1 is the firm specific risk premium calculated using results from regression one and subtracting ERP and Treasury rate. EquityPurchased is the % of equity PE purchased for the surveyed investments. EquityofWACC is the % of equity used in WACC. The regression is run using Newey standard errors. R-squared is 11.14%. %, †significant at 10%, ∗significant at 5%; ∗∗significant at 1%.

$$FSRP\ 1_{ij,t} = \beta_0 + \beta_1 EquityP\ urchased + \beta_2 Equityof\ WACC + \epsilon_{ij,t.}$$

Table 15: Estimated FSRP1 by WACC & Equity Purchased

WACC Equity	Equity Purchased							
WACC Equity	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	2.98%	3.52%	4.05%	4.58%	5.11%	5.64%	6.18%	6.71%
0.1	2.67%	3.21%	3.74%	4.27%	4.80%	5.33%	5.87%	6.40%
0.2	2.37%	2.90%	3.43%	3.96%	4.49%	5.03%	5.56%	6.09%
0.3	2.06%	2.59%	3.12%	3.65%	4.18%	4.72%	5.25%	5.78%
0.4	1.75%	2.28%	2.81%	3.34%	3.88%	4.41%	4.94%	5.47%
0.5	1.44%	1.97%	2.50%	3.03%	3.57%	4.10%	4.63%	5.16%
0.6	1.13%	1.66%	2.19%	2.72%	3.26%	3.79%	4.32%	4.85%
0.7	0.82%	1.35%	1.88%	2.42%	2.95%	3.48%	4.01%	4.54%
0.8	0.51%	1.04%	1.57%	2.11%	2.64%	3.17%	3.70%	4.24%
0.9	0.20%	0.73%	1.27%	1.80%	2.33%	2.86%	3.39%	3.93%
1	-0.11%	0.42%	0.96%	1.49%	2.02%	2.55%	3.08%	3.62%

Note.-Table 15 displays various estimated IRR residuals from experiment 1 - regression 2 results. *WACC Equity* is the amount of equity used vs. debt by the PE firm and *Equity Purchased* is the average amount of equity purchased by the PE firm in the portfolio companies. The IRR residuals are calculated using the coefficients from shown in Table 14.