A Note On Using Regression Models to Predict the Marketability Discount

by

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Summary

Academic research suggests that the size of the marketability discount is in the neighborhood of 13.5% and more recent work has suggested it may be as low as 7.23%. The regression models of Silber, and Hertzel and Smith have provided both the intellectual and empirical basis for these conclusions. These models were initially developed to study the determinants of the marketability discount. It has been suggested that they should now be used as a basis for forecasting the marketability discount. This paper demonstrates that these models should not be used for this purpose because the forecast errors are likely to be large. Moreover, based on the structure of these models and their prediction errors, it is not possible to state with any certainty that a 13.5% marketability discount is statistically different than a discount of say 25%.

I. The Problem

Past issues of the *Business Valuation Review* contain several articles that contribute to our understanding of the issues involved in quantifying the magnitude of the marketability discount. However, its size and an accepted methodology to quantify it are still very much open questions.[i] Kania's recent review of several marketability discount studies has correctly framed the debate on these issues.[ii] He notes that peer reviewed finance research based on regression models of the marketability discount support the following conclusions:

- The regression models of Silber (S) and Herzel and Smith (HS)[iii], the research most often cited in the context of the marketability discount, support the hypothesis that there are independent factors that determine the size of the discount.
- Simply applying a median or average marketability discount without considering the factors that determine each yields predictions of the discount that are likely to be incorrect.
- "In addition to explaining the size of the total marketability discount by multiple causal factors, such a model estimated with a regression methodology has the added benefit of being able to objectively predict a discount of a closely held firm."[iv]

While regression modeling can be useful in predicting the size of the marketability discount, the S and HS models cannot be used for this purpose. These models were estimated for the sole purpose of testing hypotheses about the size and statistical significance of variables suggested by finance theory to determine the size of the marketability discount. Models built for this purpose are not required to pass the more rigorous standards of performance that models built to predict must meet. To my knowledge, no such testing has occurred. But even if it has, a cursory review of the summary statistics of both models indicates that a valuation analyst could not be confident that either model would yield an accurate prediction of the marketability discount for a firm included in the estimation sample and even less certain that these models can produce an accurate forecast of the marketability discount for a firm not included in either sample.

This paper will demonstrate that any point estimate prediction of the marketability discount from either the S or HS model is accompanied by a confidence interval within which the "true marketability discount" is expected to lie that is very wide. This means, for example, that a prediction of 13.5% for the discount and a prediction of 25% may not be statistically different. This point is very important since the debate about the size of the discount is really about whether the values often used, 20%+, are significantly larger, in a statistical sense, then the values indicated by HS (13.5%) and Baj etal. (7.23%) who re-estimated the HS model with more current data.[v] Moreover, the cross-sectional nature of the equations would likely limit their ability to predict the size of the marketability discount within a sufficiently narrow range for firms not included in the estimation sample. This occurs for two reasons. The first is that the firm for which a prediction is needed may have characteristics that are significantly different than those firms that were part of the estimation samples used by S and HS to estimate their models.[vi] Thus, for the firm in question, we cannot be sure that the coefficients of the regression models adequately reflect the relationship between the marketability discount and the variables that determine it. Second, the coefficients of these models have not been shown to be time invariant, which is always a concern when using cross-section regression models to make predictions outside the time period over which the models were estimated. This last point is important because the time period for which the discount prediction is needed is usually far removed from the time interval over which the regression models were estimated. Thus, if the coefficients are found to vary by time period and by characteristics of firms included in the sample, then the models could not be counted on to produce out-of-sample predictions that are credible.

II. Cross- Section Models of the Marketability Discount

The marketability discount models of S and HS are appealing because they explicitly relate firm-specific variables to the size of the marketability discount. In so doing, they formally recognize that simply applying a median or average marketability discount without controlling for the other factors that may impact the size of the discount would result in significant prediction error. Analysts have asserted for some time that the variability in reported average and median discounts may be a function of measuring the discount without controlling for these other variables.[vii]

The central purpose of the work of S and HS was not to develop models that predict the size of the discount but rather to test whether certain determinants of the discount suggested by finance theory were in fact supported by the data. The firm is the unit of observation and hence the regression models are estimated across firms during an interval of time. The S and HS models are reproduced below.

Silber Cross-Section Model of Restricted Stock Discount

$$LN(P^r/P) = \underset{(.13)}{4.33} + \underset{(.013)^1}{.036}*LN(REV) - \underset{(.051)^1}{.142}*LN(RBT) + \underset{(.108)}{.174}*DERN + \underset{(.154)^1}{.332}*(DCUST)$$

R^2 = .29; Standard Error of Regression = .358; F= 8.1; [1] = coefficient statistically significant; Variable Names:1) REV= firm revenues, 2) RBT = restricted block to total shares outstanding, 3) DERN = dummy variable equal to 1 if earnings are positive, 0 otherwise, 4) DCUST = dummy variable = 1 if there is a customer relationship between the investor and the firm issuing the restricted stock, 0 otherwise.

Time Interval: 1981- 1988; Data: Security Data Corporation: 69 private placements of common stock of publicly traded companies

Hertzel and Smith Cross-section Model of the Private Placement Discount

% Discount = 1.4^1+ .41*Fraction Placed^1 +.091*Financial Distress – .141*BME^1

- .078*Log of Proceeds^1 + .135*Restricted Shares^1 -.091*Single Investor + .021*Management Buyer;

Adjusted R^2 = .413; F = 11.55; ^{1} = coefficient statistically significant; Variable Names: 1) Fraction Placed = shares placed to total outstanding after placement, 2) Financial Distress =dummy variable = to 1 if firm was facing financial distress, 0 otherwise, 3) BME = book to market value of equity, 4) Proceeds = $ value of placement, 5) Restricted Shares = dummy variable = to 1 if private placement was restricted, 0 otherwise ,6) Single Investor = dummy variable equal to one if only one investor purchased placement, 0 otherwise, 7) Management Buyer =dummy variable = to 1 if investor was a manager of the issuing firm, 0 otherwise
Time Interval: 1/1/80 to 5/31/87: 106 private placements

Econometricians refer to these models as cross-section models. They are designed to test the hypothesis that the explanatory variables impact the marketability discount for each firm in the sample in precisely the same way during a selected time interval. This appears to be the case here, but upon further analysis one uncovers important limiting factors that preclude the use of either model as a predictive tool.

1. For both models, the explanatory power is quite weak. The R^2 of S is less than 30%, and the adjusted R^2 for HS is a bit over 41%. This means that the unexplained variance of the marketability discount exceeds the variance that is explained by the model. There are multiple possibilities as to why this is the case, but the central point is that both models have limited explanatory power. In the case of S, the standard error of the regression is .358, which indicates that the in-sample prediction interval from the model can be enormous. S ran several simulations with his model. He states: "Table III uses Equation (6) to estimate how the price penalty varies with revenues and block size when firms have positive earnings and sell restricted stock to an unrelated party."[viii] When a hypothetical firm has revenues of $40 million and places a restricted block size of 13%, the model predicts a discount of 28.5%. Based on the standard error of the equation, we could be 68% certain that the actual discount will lie within a confidence interval bound by a premium of 2.5% (28.5% plus one standard error) on the low side and a discount of

50% (28.5% minus one standard error) on the high side.[ix] If we desire more certainty, the prediction interval becomes much larger. HS do not provide sufficient information to make the same assessment, but given the explanatory power of their equation, the likelihood is that the band of certainty around any in-sample point estimate would also be unacceptably large.[x] Finally, if the within sample prediction errors are large, there is absolutely no reason to think that the out-of-sample prediction errors will not be equally as large and unacceptable. However, as we show below, even when a regression model explains over 90% of a dependent variable's variation, the out-of-sample prediction error can still be unacceptably large.

2. Predictive accuracy of the estimated relationships is not the central concern of S or HS. Rather, the researchers were only interested in testing various hypotheses and measuring whether the coefficients of the estimated models were statistically significant. However, statistical significance only means that the coefficient is not likely to be zero. It does not mean that the "true" value does not lie within a very wide and perhaps, from the vantage point of prediction, unacceptably wide range defined by the estimated coefficient and its standard error. For example, the coefficient of LN (RBRT) in the S model is -.142 and is statistically significant. Given its standard error of .051, we could be 68% certain that the "true" coefficient lies within an interval bounded by -.09 (-.142 + .051) on the high side and by -.192 (-.142 -.051) on the low side. Assuming for the moment that RBT is 13% and other variables in the S model have zero values, this implies that the marketability discount could be 40 % if the coefficient were -.09 or 53% if the coefficient were -.192. Keep in mind that although the marketability discount prediction interval is wide, both values are still consistent with the model's estimated structure.

3. The above example is relevant to the size of the marketability discount directly estimated by HS. The interpretation given to the coefficient of the restricted stock variable, 13.5%, is that it represents the discount investors require when purchasing illiquid restricted

stock. Kania has argued that this implies that when other factors are controlled for, the marketability discount is indeed much lower than the 20% plus values that are often used when valuing illiquid stock.[xi] Again, while this may be true, it may also be the case that the true coefficient may lie within a wide range and that a 20% discount or more may also be consistent with the HS model. The authors do not report coefficient standard errors so one cannot make this determination. But based on test statistics they do provide, one would not be surprised if the standard error of the restricted stock coefficient were large enough to suggest that the "true coefficient" lies within a band that includes 13.5% and 20%.[xii] [xiii]

4. A key underlying assumption of any regression model is that its error variance is constant for all observations. Cross-section models of the S and HS type generally do not meet this condition, and unless it can be corrected, tests of hypothesis are not meaningful.[xiv] HS do adjust their hypothesis testing procedures to handle this problem.[xv] S does not. Regardless of whether the hypothesis testing procedures have been adjusted or not, the parameters of the estimated models have not been corrected for this problem.[xvi] This means that the standard errors of the estimated coefficients are estimated incorrectly, which in turn means that the forecast interval implied by these estimates is also incorrect. Thus even if the models are "true", that is they include the correct explanatory variables, using the equations to make out-of-sample predictions of the marketability discount will yield a prediction interval within which the "true marketability discount is expected to lie that is incorrect. As we show in the Appendix, the marketability discount confidence interval is likely to be smaller than it should be for target firms that are smaller than the firms used to estimate the S and HS models. This means that a third party using the statistical information from either the S or HS models to determine the size of the confidence interval, like the IRS for example, may conclude that the discount used is

statistically different than the benchmark selected by the third party, e.g. 13.5%, when in fact it is not.

5. In cases when equations contain statistically significant coefficients and low explanatory power one is inclined to conclude that the models are unstable and thus produce unreliable predictions. There are many causes of model instability, but one that characterizes cross-section models is left out explanatory variables. That is, important determinants of the dependent variable have not been identified, or if they have, they have not been included in the model by researchers. These left out variables do not necessarily bias the tests of hypothesis, but they do impact the reliability of the out-of-sample forecasts of the model.[xvii]
6. What might a plausible left out variable be? There may be more, but one that immediately comes to mind is the interest rate level prevailing at the time a firm's marketability discount was calculated. The sample periods covered by HS and S is 1/1/80 – 5/31/87 and 1980-1988 respectively. The marketability discount for each firm was calculated at a discrete point in time over the respective sample periods. At each measurement point, the interest rate level was different. These differences were not included as variables in either the S or HS models. We know that interest rates declined precipitously from 1980 to 1988, which is consistent with the easing of credit conditions and a greater availability of liquidity[xviii]. Thus, it would not be surprising that under these conditions we found the marketability discount to be lower for privately placed and/or restricted equity at the end of the sample period than at the beginning. If so, then this implies that if a firm placed private or restricted equity at both the beginning and at end of the sample period, then the discount would be smaller for the later than the earlier placement all else equal. Because the interest rate influence would be effectively averaged by the regression using the HS or S equations as they stand, predictions of the marketability discount would be lower than they should be during periods when interest

rates were above the average level prevailing during the estimation period and greater than they should be during periods when interest rates were below the average.

III. The Cross Section Time Series Conundrum

The potential for the level of interest rates to impact the size of the discount underscores the age-old problem of using a cross-section model in a time series context. The reason is that by construction a cross-section model only applies to the time interval of estimation. In order to apply the relationship to a firm in say a later time interval, one would have to show that the coefficients of the cross-section models are time invariant. Put differently, a model estimated with one sample of firms would have to demonstrate that it could accurately predict values of the dependent variable in another sample at a different point in time before one could conclude that the model's predictive characteristics were satisfactory. S or HS offer no evidence that their estimated models are temporally stable. This is not a criticism of their work. It is simply a limitation of their research design. In both instances, the authors wanted to test hypotheses about the determinants of the marketability discount and did not want to test whether the models were robust enough to be used as predictive tools.

An Example Demonstrates the Dilemma:

To understand the implications of using a cross-section model inappropriately, I appeal to research conducted by Damodaran on the cross-section instability of models that explain a firm's price-earnings (PE) ratio, a variable not unlike the marketability discount in terms of its firm-specific variability.[xix] Damodaran extracted information on PE ratios, payout ratios, earnings growth rates and betas for all NYSE and AMEX listed firms over the 1987 to 1995 period. He then ran a cross-section regression for each year where each firm's PE was related to the variables noted below.

1987: PE = 7.189 + 13.05*Payout - .6259*Beta + 6.5659*EGR; **R^2** = .9287

1988: PE = 2.5848 + 29.91*Payout – 4.5157*Beta + 19.9143*EGR; **R^2** = .9465

1989: PE = 4.6122 + 59.74*Payout - .7546*Beta + 9.0072*EGR; **R^2** =.5613

1990: PE = 3.5955 + 10.88*Payout - .2801*Beta + 5.4573*EGR; **R^2** = .3497

1995: PE = 19.77 – 12.54*Payout - .4122*Beta + 15*EGR; **R^2**= .2794

- PE = end of year price earnings ratio
- Payout = percent of net income paid as dividends at end of year
- Beta = beta of stock estimated by using returns over previous five years
- EGR = expected earnings growth over the next five years

There are several important things to notice about these equations that have relevance to the structural stability of the marketability discount models.

1. Just because an equation has a high R^2, it does not follow that the estimated relationship is stable. This point is underscored by the fact that the R^2s or explanatory power of the PE equations were high in 1987 and 1988; and the relationship completely broke down in 1989, 1990, and 1995.
2. The above structural instability is also accompanied by estimated coefficients of the PE equations that vary a great deal from one year to the next. In 1995, for example, the coefficient of the payout variable is negative while it is positive in each of the other years. Coefficient variability of this sort indicates that the relationship between the payout ratio and the PE variable changes over time and that this variability is not captured by these cross-section PE models.

IV. Marketability Discount Regression Models Cannot Be Used as They Stand

Based on the above discussion, it is clear that the use of either the S or HS models to make out-of-sample predictions of the marketability discount is filled with potential landmines. Prior to using either model to predict the marketability discount, they should be properly re-estimated and then a formal analysis of their out-of-sample prediction errors should be carried out. Based on this out-of- sample analysis, the standard errors of the out-of-sample predictions can be calculated and evaluated. At this point either model can be used to make a point estimate prediction of the marketability discount for a target firm. This, combined with the previous analysis of prediction errors, can then form the basis for a confidence interval within which the "true marketability discount" is expected to lie. Once the range is established, the choice of the exact marketability discount to apply could then be based on the unique circumstances of the firm in question and on informed analyst judgment. This approach would allow the analyst to apply a discount that is different from either model's point estimate yet use a value that is fully consistent with the structure of the regression model used to develop the confidence interval.

Because the above requisite information is not currently available, I would be very cautious using either model to predict the size of the marketability discount. While the S and HS models represent critical first steps towards understanding the determinants of the marketability discount, there is good deal of research that still needs to be done before these models can be used as credible prediction tools.

V. Conclusion

It may seem to some that issues raised here are quite technical and are in an area that is somewhat removed from traditional business valuation issues. The problem is that more practioners are turning to the use of regression models to develop predictions of important valuation variables like the marketability discount. Applying regression techniques to a valuation issue often gives it a veneer of correctness that may be undeserved. We have shown that this is likely to be the case for regression models of the marketability discount that were designed to test hypotheses and not designed as prediction tools. Using such models incorrectly may result in large unintended prediction errors. At the moment the size of these errors is not known, but given the summary statistics of the regression models customarily referenced, it is quite likely they are large. This means that the range within which the "true the marketability discount" lies for any target firm is likely to be quite wide. Therefore, applying a discount that appears to be significantly different than some established benchmark, say 13.5%, may in fact be perfectly consistent with the benchmark based on known statistical properties of the underlying models that have given rise to the benchmark to begin with. This result may not be satisfying, but it is the best we can do until the predictive properties of marketability discount models have been studied further and more powerful model constructs have been put forward.

Appendix

As noted in the text, when the in-sample error variance is not constant, this condition will also be true for the out-of-sample forecast error variance. This means that the estimated forecast interval will be larger or smaller than the true confidence interval. Research indicates that the marketability discount is inversely related to firm size.[xx] Thus, one might expect that the errors in forecasting the marketability discount are greater for smaller, less well-known firms than for their larger better-known counterparts. This implies that the variance of the forecast errors for smaller firms is greater than the variance of forecast errors for larger firms. To see the implications of this, let us consider a general formulation of the S and HS models.

$$y_i = a_0 + b * x_i + e_i$$

$where:$

$y_i = \text{marketability discount for firm } i$

$x_i = \text{a column vector of variables that determine the marketability discount}$

$b_i = \text{a row vector of coefficients that measure the influence of each varaiable on the marketability discount}$

$e_i = \text{the error in predicting the marketability discount}$

Let us assume that S or HS did an out-of- sample forecast for a set of firms that had the same general characteristics as the sample of firms used to estimate the parameters of these models. Based on information from the S study, firm revenue ranged from a minimum of zero to a maximum of \$595 million with a median of \$40 million. Though HS do not provide similar statistics, from the information they do present, the firms in their sample appear to be somewhat larger than those in the S sample.[xxi] Let us further assume that as part of their analysis, the researchers calculated the variance of the forecast errors from their models. This variance is defined below.

$$\sigma_f^2 = \sigma_i^2 + \sigma_y^2; \quad \sigma_i^2 = \sigma_e^2 \ \ \textit{for all } i$$

where :

$\sigma_f^2 =$ *forecast error* var*iance*

$\sigma_e^2 =$ var*iance due to random disturbanc e*

$\sigma_y^2 =$ var*iance due to sampling error and is related to fact that the forecast sample is small relative to the size of the population. As sample size increases,* σ_y^2 *approaches zero.*

Since the forecast error variance varies with the size of the firm, σ_f^2, is not constant. This occurs because $\sigma_i^2 \neq \sigma_e^2$. A formal representation of this is shown below.

$$\sigma_i^2 = \{(S_i)^{-c}\} * \sigma_e^2;$$

$S_i =$ *sales of firm i*; $c =$ deg*ree of heteroskad asticity*

where $c = 0$ *indicates that the* var*iance of the random errors is cons* tan *t*.

Substituti ng the above exp*ression for* σ_i^2 *in the* σ_f^2 *relationsh ip yields* :

$$\sigma_f^2 = \{(S_i)\}^{-c} \sigma_e^2 + \sigma_y^2$$

$$\sigma_f = [\{(S_i)\}^{-c} * \sigma_e^2 + \sigma_y^2]^{1/2} = s \tan \textit{dard error of the}$$

out – *of* – *sample forecasts*.

Now based on σ_f and a marketability discount benchmark, MDB, selected by an official third party, assume the third party establishes an official confidence interval within which it expects the true marketability discount to lie. Assume that the interval selected is bounded on either side by one standard error. If the errors are normally distributed, we can be 68% certain that the "true" marketability discount lies within the confidence interval shown below.

$$1\sigma_f \text{ - MDB } + 1\sigma_f$$

Let us assume the official third party chose a MDB equal to 13.5% and a valuation analyst used the S or HS models to develop a marketability discount of 25% for a $10 million revenue firm. If σ_f is estimated to be 10%, then the official third party would conclude that a marketability discount of 25% is outside the confidence interval- (13.5% + 10%< 25%). However, this conclusion would likely be wrong. We know this because the forecast errors used to calculate σ_f are heteroskadastic. They vary inversely with size of the firms in the forecast sample and we know based on the data provided by S that the firms in the sample are much larger than the $10 million target firm. Since this is the case, we know that σ_f would be larger if the underlying forecast sample were either made up of firms of similar size to the target or the size bias was corrected for when the forecast model was initially estimated. Let us assume that the size bias was corrected for and as a result σ_f was calculated to be 12% instead of 10%. In this instance the official third party would conclude that the 25% marketability discount forecast was in the "range of acceptability" since (13.5% + 12%) < 25%.

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End Notes

[i] Kania, John, "Marketability Discounts Are Lower Than Many Appraisers Believe- A Reply to Shannon Pratt's Critique of My Marketability Discount Studies, Letter to Editor, *Business Valuation Review*, December 2001, pp.56-59

[ii] Kania, John, "Has Restricted Stock Data Become Irrelevant for Determining Marketability Discounts?", *Business Valuation Review*, Volume 20, No. 2, June 2001, pp. 10-14.

[iii] Hertzel, Michael and Smith L. Richard, "Market Discounts and Shareholder Gains for Placing Equity Privately", *The Journal of Finance*, Vol. XLVIII, No.2, June 1993, pp.459-485.

Silber, William, Discounts on Restricted Stock: The Impact of Illiquidity on Stock Prices, *Financial Analysts Journal*, July-August 1991, pp.60-64

[iv] Kania, John, "Evolution of the Discount for Lack of Marketability", *Business Valuation Review*, March 2001, p. 14.

[v] Bajaj, Mukesh, Denis, David, Ferris, Stephen and Sarin, Atula, "Firm Value and Marketability Discounts", working paper.

[vi] Using regression models to predict an event outside the experience over which the models were estimated yields predictions that are likely to be biased and result in drawing incorrect inferences.

[vii] Koeplin, John, Sarin, Atulya, Shapiro,Alan, "The Private Company Discount", *Journal of Applied Corporate Finance*, Winter 2000, Volume 12, Number 4, pp. 94-101

[viii] Silber, p.64.

[ix] Using the Silber equation and the values in the text yields a prediction for Ln(RPRS) of 4.27. The antilog of this value is 71.52. This value less a 100 is the size of the discount, which is 28.5%. Therefore, 4.27 +.358(one standard error) is 4.63. Its antilog is 102.5, which is a premium of 2.5%. Following the same procedure, 4.27-.358 is 3.91 or a discount of 50%.

[x] I have e-mailed and called Professor Hertzel to obtain the data set underlying the HS model but to this point there has been no response.

[xi] Kania, John, "Marketability Discounts are Lower Than Many Appraisers Believe- A Reply to Shannon Pratt's Critique of my Marketability Discount Studies", Letters to the Editor, *Business Valuation Review*, December, 2001, pp.56-59

[xii] The author has requested the standard errors of the estimated coefficients but HS have not replied.

[xiii] A version of the HS model has been estimated by Bjaj et al. The authors run a simple simulation that suggests that a discount can be as low as 7.23%. Kania references this conclusion in his rebuttal to Pratt. The problem is that the Bajaj model does not explain any more of the variation in the marketability discount than the HS model does and thus the true marketability discount is likely to be within a wide interval centered on any marketability point prediction. See Bajaj

[xiv] On this point see any econometric text e.g. Kementa, *Elements of Econometrics*

[xv] The authors adjust coefficient standard errors using White's correction so they can conduct meaningful tests of hypothesis. This approach merely corrects for the bias inherent in the original estimation. It does correct the estimated standard errors of the coefficients that reflect the bias.

[xvi] HS employs the White correction to ensure that tests of significance of model coefficients are not biased in the presence of error variances that are not constant across observations. This statistical problem is known as heteroskedasticity and it characterizes virtually all cross- section models. This means that if we proceed with the regression analysis without correcting for this problem, any inference about the size of the "true coefficients" will be incorrect- that is, the calculated confidence intervals and acceptance regions will be wider or narrower than the correct ones. Since correcting a cross-section model for heteroskedasticity is quite complex and because the White procedure still allows the researcher to make tests of significance in the presence of heteroskedastcity, researchers generally do not go through the time and effort to make the required corrections to a model's parameters. The upshot is that when the model is used as a predictive tool, the forecast confidence interval will be wider or narrower than the correct interval. Since the interval within which the "true" marketability discount lies is at issue, using either the HS or S model without correcting for heteroskedasticity would necessarily yield confidence intervals that are wrong. This might

result in accepting the hypothesis that an estimate of the marketability discount from either model is significantly different from an established standard, say 13.5%, when in fact it may not be true.

[xvii] If the left out variables are statistically independent of the included variables, then tests of hypothesis on the coefficients of the former variable can be meaningfully conducted.

[xviii] Between 1/80 and 12/88 the one year treasury bill rate declined from 11.04% to 8.49%.

[xix] Damodaran, Aswath, *Investment Valuation*, Wiley, 1996, p. 306

[xx] Silber, states: “The last three rows of Table II show that firms with higher revenues, earnings and market capitalizations are associated with lower discounts”(p 61).

[xxi] Silber, page p. 61; Hertzel and Smith, p.471.