Note on Using Option Pricing Models to Estimate the Value of Control

by

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I. Introduction

An acquiring firm often pays more for the target than the present value of a target's cash flows. This additional consideration is termed a control premium. When the buyers and sellers are known, and particularly when the firms involved are public, analysts generally have sufficient information to accurately estimate the size of the control premium. Because there is no organized market for private firms and transactions occur sporadically, it is often difficult for a valuation analyst to identify potential buyers. In these circumstances, the valuation analyst often uses the most recent mean or median from published control premium studies as his best estimate since the information he would need to obtain a more informed estimate, namely who the buyers are, may not be available. However, as we show below, defaulting to using the median control premium is likely to be inappropriate, and, in general, will overstate the size of the control premium and hence, the estimated the control value of the private firm. In these cases we show that the value of "pure control", the incremental value a buyer will pay to run the firm in the same way as the seller, can be estimated using an option-pricing framework. This value will be lower than the value of control that includes an estimate of the synergy that a known buyer expects to create, post transaction. This latter value can only be estimated if the buyers and/or their buying motivations are known with some degree of certainty. When this is not the case, there is no basis for estimating the synergy value and, in general, a control premium that includes it will overstate the value of control in these circumstances.

This paper first develops the concept of control as an option-pricing problem and then uses the well-known Black-Scholes option-pricing model to estimate the control premium. Put differently, this paper establishes that reported control premiums are made of two components: the value of pure control and the value of the acquirer's synergy option. We show that the former can be estimated without knowing the characteristics and/or the motives of the acquirer while the second component is a function of the acquirer's planned strategy once the target firm's assets are in its control.

II. The Control Premium Puzzle

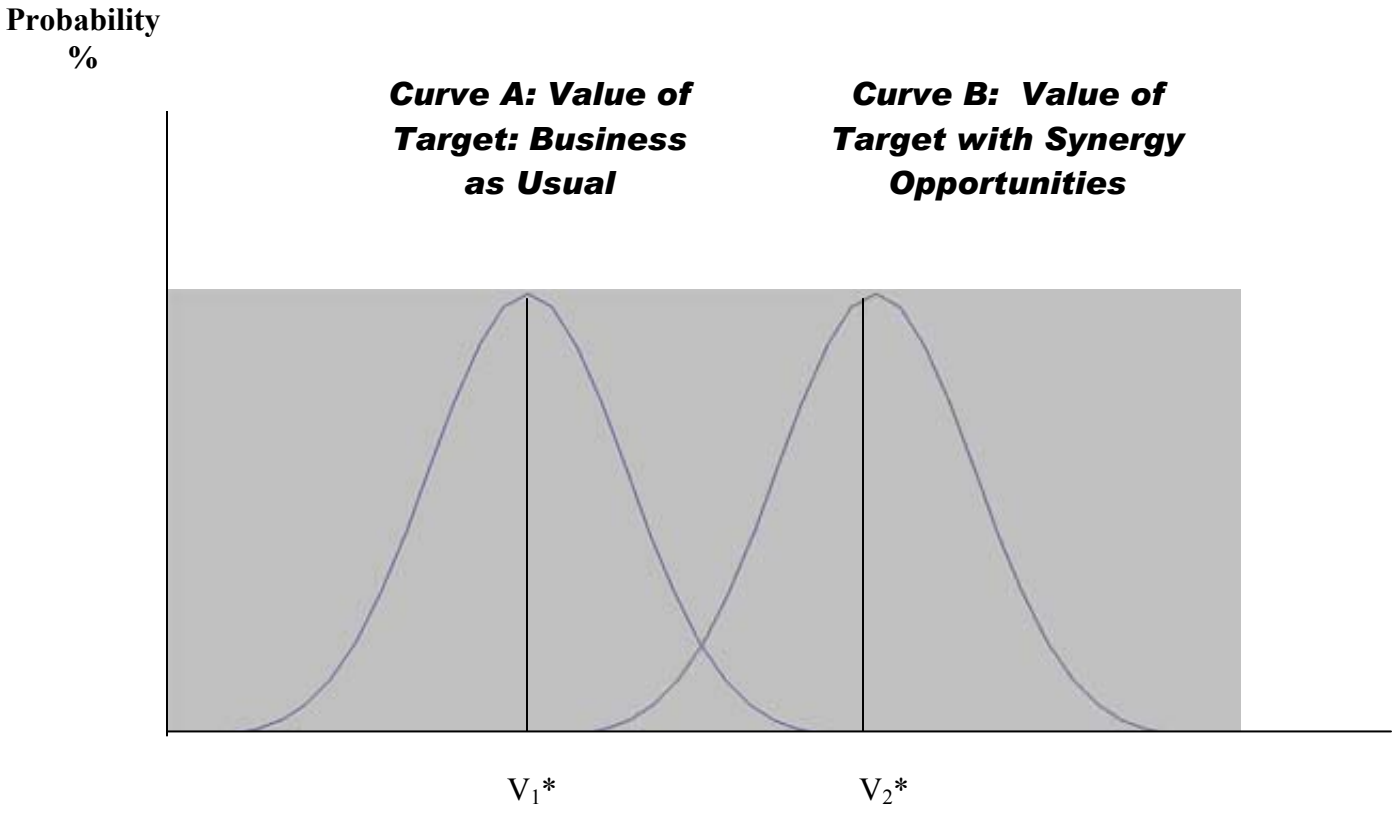
In their control premium study Houlihan Lokey Howard and Zukin define a control premium as the additional consideration that an investor would pay over a marketable minority equity value (i.e. The Wall Street Journal price) in order to own a controlling interest in the common stock of a company.[i] The authors further state[ii]:

> A controlling interest is considered to have a greater value than a minority interest because of the purchaser's ability to effect changes in the overall business structure and to influence business policies. Control premiums can vary greatly. Factors affecting the magnitude of a given control premium include:
>
> 1. The nature and magnitude of non-operating assets.
> 2. The nature and magnitude of discretionary expenses.
> 3. The perceived quality of existing management.

4. The nature and magnitude of business opportunities, which are not currently being exploited.
5. The ability to integrate the acquiree into the acquiror's business or distribution channels

The factors noted above fall into two broad categories: 1) category 1- managing the cash flows and associated assets of a target business on a business as usual basis- items 1-3- and category 2 -putting additional assets in place to take advantage of perceived business growth opportunities that are not being exploited-items 4-5. Business as usual means that management expects to run the firm in the future as it has in the past. Category 1 is distinguished from category 2 in that the former is only a function of the risks and opportunities of the business as it is currently configured. In contrast, category 2 requires the purchase of new assets to take advantage of new perceived business opportunities that have a risk and opportunity profile that are substantively different than the risks and opportunities that are inherent in the pursuit of a business as usual strategy. Category 2 requires new investment to take advantage of these opportunities, which only emerge if the target is acquired. Moreover, one can only assess category 2 factors if the acquiring firms and their strategies are known with some acceptable level of certainty. By contrast, category 1 risks and opportunities are known since they are a function only of the target firm's in-place business strategies. To see the difference between the valuation implications of category 1 and category 2 factors, consider the value distribution curves in Exhibit I.

Exhibit I: Target Firm Value Distribution Curves



Category 1 factors determine the shape of the distribution of possible valuation outcomes, curve A, with V_1* the median of the distribution of outcomes. For purely exposition purposes, we assume the value distribution is normal. The curve shows that a business as usual strategy can give rise to a multitude of valuation outcomes, although the range of outcomes is bounded. For example, the chances of a business as usual strategy creating a value as large as V_2* is zero. However, V_2* becomes possible if the value distribution were curve B rather than curve A. However, curve B is only possible when category 2 factors are in play. That is, category 2 factors are different in that they are a function of buyer's capacity to alter the shape and/or position of the target firm's distribution of valuation outcomes. Here, the probabilities associated with different valuation outcomes are only known when buyers both declare themselves and provide sufficient data to allow one to make a judgment about various valuation outcomes. Category 2 related outcomes are not possible when the target adopts a business as usual strategy. They only emerge when the assets of the target and the buying firm are joined creating the potential for new possibilities. We refer to this co-joining of assets as synergy options. Based on this articulation, we assert that a control premium is made up of two components: the value of pure control and the value of synergy options.

This assertion provides the logic, and as we show below, the mathematics for establishing a theoretical range for the control premium. For example, if the market of buyers is made up of those that will generally manage the business in much the same way as it has been managed, then one would conclude that the control premium paid should not exceed the value of pure control. As a practical matter, market conditions at the time of the

transaction will dictate whether the winning bid will include a control premium that is above or below the value of pure control. However, we would expect the average of these deviations to be zero across a sufficient number of non-synergy transactions. We would also expect a similar outcome when the buyers have synergy options. Thus, we argue that the expected value of any control premium is equal to the expected value of pure control plus the expected value of the synergy option. Although acquirers will pay premiums outside this range, deviations should be limited by the gravitational pull of any established control premium range.

The control option-pricing framework offers several important insights into the control premium puzzle. First, the value of pure control implies that even if a buyer plans to continue a business as usual strategy and manages the assets in the same way as the current owner, the buyer would be willing to pay a premium over the present value of cash flows. Why? The answer is that there is always a chance that circumstances will emerge where the value of a firm's assets will be further down the right-hand side of the value distribution. The premium paid is the cost incurred for the right to be able to capture this benefit if it occurs. Hence, one can think of a two- stage transaction process. In the first, the acquirer buys a pure control option from the seller with an exercise price equal to the minority value of the firm. The buyer retains the right to exercise for some pre-determined period. During stage 2 the buyer decides whether to exercise or not. If the buyer exercises, then the price paid for the firm is equal to the firm's minority value, the present value of expected cash flows, plus the price of the control option.

The second implication is that the value of pure control can be determined without knowing who the buyer happens to be since its value only depends on the risks and opportunities inherent in the business as usual activities of the selling firm. Third, as a practical matter, many private firm valuations are done where the buyers are either not known or if they are their motives for purchasing are not well understood by the valuation analyst. This occurs because private firm transactions are discontinuous and information required to understand the motives of buyers is not publicly available. Hence, the costs of acquiring this information are prohibitive. In this circumstance, any control value applied by the analyst should only reflect the value of pure control.

This last point has very practical implications for how controlling and minority interests are valued. It is quite common that when a valuation analyst is valuing a controlled transaction, the explicit premium applied is an average or the median of control values from a current control premium study.[iii] Often the valuation analyst looks for guidance from past court decisions or perhaps the IRS has opined on an allowable control premium range. However, reliance on these sources should not provide the valuation analyst with a sense of comfort since the logic embedded in such solutions are not, except by chance, consistent with what the premium would in fact be if a transaction took place. Buyers and sellers establish these premiums based on the unique characteristics of the assets being transacted and what the buyer plans to do with the assets once owned. Hence, any estimate of what the proper control premium ought to be should be the result of quantitatively linking the risks and opportunities inherent in the transaction to the size of

the expected premium paid. Defaulting to applying a median control value does not meet this standard.

III. The Value of Pure Control: Setting the Stage

Let us consider the case of the purchase of a local veterinary practice by a firm whose strategy is to "roll up" veterinary practices. The "roll up" strategy is designed to create value through the introduction of professional management, reductions in overhead costs, and significantly lower prices for supplies when they are purchased in bulk. Finally, by having a network of veterinary practices covering a wide geographic area, customers can more easily be retained by the network even though they are lost to the local practice. Hence, revenue retention is greater and the cost of obtaining new customers for any one practice in the network is necessarily lower. Based on these facts, perhaps the value of control is worth about 20% or more over any reasonable estimate of the present value of the target's cash flows.

But what happens if the strategic buyer decides not to buy any more practices and there are no other like strategic buyers willing to commit funds around the valuation date? Does this mean that a veterinary practice that just comes on the market should command no control premium? The answer is the firm's value should reflect a control premium but not the value assigned by the strategic buyer. The reason is that owner of the firm has decided to deploy the assets of the firm in a certain way in order to achieve the firm's current cash flow status. The control owner has the "right" to change the way the firm's assets are deployed and can do this at his/her discretion. This is what is meant by control;

having the right to change the way the assets of the firm are used and/or financed. This right has value no matter who the potential buyer is.

To see these points more clearly, let us consider the following hypothetical. Let us assume the control owner has a portfolio that is made up of the value of the cash flows from current assets and a control option on these assets. The owner desires to sell the business and the buyer indicates he/she is willing to purchase it at a price equal to sum of the present value of the expected cash flows, although the buyer needs some additional time to evaluate whether the firm has additional cash flow potential that is not reflected in the selling price. The seller indicates that he will sell the buyer a call option on the firm with an exercise price equal to the present value of expected cash flows. The option can be exercised at anytime over the course of the next twelve months. The buyer agrees and subsequently exercises the option and purchases the firm. The purchase price, which is the firm's control value, is then equal to the present value of the expected cash flows plus the price of the call option. In this setting, the present value of expected cash flows is equivalent to a firm's minority value since this is what a rational investor would pay for these cash flows. The call option is exercised when the buyer believes that current owner will not be able to deliver the expected cash flows that are the basis for determining the firm's minority value. Thus, the call price reflects the value the buyer places on control. The seller, on the other hand, receives incremental cash equal to the price of the control option prior to the sale of the firm, which can be reinvested and earn a return in perpetuity.

Before we turn to the issue of how much above pure control a potential buyer might pay, that is the value of what we term the synergy option, let us consider the issue of pure control from another perspective. Let us assume that a recent veterinary school graduate desired to only purchase the cash flow of the veterinary practice. The current owner retained control and agreed to remain and carry on his veterinarian duties in return for receiving a market wage. In return for a one-time payment of $100, the owner agreed to distribute the cash flow of the practice to the veterinary graduate in perpetuity. This arrangement is certainly a cheaper alternative than buying a call option and then exercising it since this strategy would cost $100 plus the price of the call. But is it? What if one day the control owner decided to increase his salary such that there was no cash flow to distribute to the recent graduate? What recourse would the graduate have? The answer is clearly none. Hence, the recent graduate who wanted to purchase the veterinary practice would pay more than $100 for the practice to insure that he has sufficient control of the firm's assets and the cash flows they generate. The value of pure control is equivalent to an insurance policy that pays off when the control owner fails to deliver the promised cash flows. From the seller's perspective, he would accept $100 today and a promise to deliver future cash flows to the buyer or charge the buyer an increment over the $100 that would convert this promise to a contractual guarantee to turn control over to the buyer if the seller directed cash flow payments to himself that violated specific agreed upon guidelines. A rational seller would certainly charge the buyer something for this guarantee, and a rational buyer would pay it.

IV. The Synergy Control Option

The synergy control option emerges when a potential control buyer expects to deploy the assets of the target firm in a way that attempts to exploit new business opportunities and or integrate the target's assets with those of the acquirer to obtain cash flow benefits that were not possible in absence of the combination. This incremental cash flow results in a greater value for the control buyer and thus he is willing to pay a premium above the value of pure control because the expected value possibilities are now far greater than they were when the business was a standalone operation.

To see why this is so, let us return to the veterinary practice example and assume that a strategic buyer who owns several upscale veterinary practices that are advertised as "dog hotels" is interested in purchasing the practice. The current owner houses and cares for dogs in the traditional way. The buyer believes that by combining the target practice with those that the strategic buyer already owns will enable him to reduce the costs of operating the target practice as well as raise prices for additional services offered by the "dog hotel". The cost synergies emerge because redundant costs can be removed when the firms are combined that could not be when the target was a standalone. Such costs include administrative costs and purchasing necessary supplies at lower unit prices because the larger entity can purchase in bulk and receive discounts that a smaller operation could not. The cost of capital will also likely be lower because a larger firm is likely to be a better credit risk than a smaller firm. In addition, creating a more upscale image will allow the strategic owner to raise prices for traditional services, which will be produced at lower costs. Profit margins will expand, and expected cash flows will

increase. Aggregating the benefits of combination, the synergy buyer believes that the firm with expected synergies could be worth as much as $200. Remember that the present value of the veterinary practice's cash flows under current management is only worth $100. To generate as much as an additional $100, the new buyer estimates that an additional $50 of investment would be required. As we show below, this synergy investment can be valued as a call option on additional firm assets.

For argument sake, let us assume that the synergy and pure control options are worth $14 and $11 respectively. What is the minimum control value the target will accept and the maximum control value the strategic buyer would be willing to pay? The minimum control value is the value of the pure control option- $11. The maximum control value is $25 of which $11 is the value of pure control and $14 is the value of the synergy option. As a practical matter, how much the strategic buyer will actually pay depends on the acquirer's bargaining power relative to the bargaining power of the target. What we know from recent studies of private firm acquisitions by public firms is that private firm targets generally have less bargaining power than their public firm acquirers.[iv] This means that private firms appear to be receiving less then they might and public firms are retaining more of the expected wealth creation that occurs as a result of the acquisition.

V. The Option-Pricing Model

In this section, we use the non-dividend paying version of the Black-Scholes option-pricing model to value each of the components of the control premium. The basic equations are shown below.

$$TCP = CP_p + CP_s$$

$$CP_j = V_0 * N(d_1) - X * e^{-rT} * N(d_2);\ j = p, s$$

$$d_1 = (\ln(V_0 / X) \quad + (r + \sigma^2 / 2) * T) / \sigma * T^{.5}$$

$$d_2 = d_1 - \sigma * T^{.5}$$

$$N(d_i) = (1/(2\pi^{.5}) \int_{-\infty}^{d_i} e^{-X^2/2}\, dX\ ,\ i = 1,2$$

where:

1. TCP is the total value of control.
2. CP_p is the value of pure control.
3. CP_s is the value of the synergy control option or the value of a call option on additional assets needed to execute the acquirer's strategy.
4. Vo is the value of the target firm's cash flows as a standalone entity.
5. T is the time to expiration of the option. T varies with the type of option being considered.
6. r is the risk free interest rate with a duration equal to T.
7. e^{-rT} is the discount factor based on continuous compounding.
8. X is the exercise price. For CP_p it is equal to V_0 and for CP_s it is equal to the investment required to create the synergy value.

9. σ is the standard deviation of returns. For CP_p it is equal to the standard deviation of returns on firm equity prior to the acquisition. For CP_s it is equal to the standard deviation of returns on equivalent synergy investments.
10. $N(d_i)$, $i = 1,2$ is the cumulative probability density function.

Valuing the Pure Control Option As demonstrated below, the value of an option increases with time to expiration and volatility of returns on the underlying assets. The reasoning is as follows. The longer the time to expiration of the option, the more time there is for the value of the underlying assets to exceed the purchase or exercise price. The greater the volatility of the returns on the firm's assets, the greater the potential of asset returns being high resulting in the market value of the underlying assets exceeding the exercise price. Since volatility is symmetric, the market value can also be below the exercise price. However, in this case the option would not be exercised, and the transaction would not take place.

The time to expiration defines the life of the option. In the case of the pure control option, one can think of time to expiration as the due diligence period at the end of which the prospective buyer either needs to decide to exercise the option, buy the firm or not. Due diligence time frames vary, but they generally do not take longer than six months, although there are cases where they extend beyond a year. The table below assumes that the maximum life of a pure control option is 12 months. The measure of volatility required by option pricing models is the standard deviation of asset returns. An

approximation to calculating the volatility of private firm returns is described in Appendix I.

Exhibit II: Value of Pure Control Premium Expressed as a Percent of the Stock Price Prior to the Acquisition Announcement
Assumptions: Exercise Price and Market Value are $100; Risk Free Rate = 2%

Time to Expiration: Months	Standard Deviations of Returns			
	25%	50%	75%	100%
3	5.19%	10.10%	14.98%	19.81%
6	7.46	14.36	21.16	27.81
9	9.25	17.64	25.85	33.78
12	10.79	20.41	29.74	38.66

Exhibit II shows that the value of the option increases with time. Option value also increases with volatility. What is the intuition here? Paying more for risk does not seem to make sense. But it does when you consider what a pure control option is. It is insurance against making a mistake. The greater the degree of uncertainty about receiving the promised cash flows from the control owner, the more one is willing to pay for insurance to find out whether entering into the bargain with the seller makes sense. If one were certain about receiving the promised cash flows, then there would be no reason to pay a premium for them. Thus, the value of pure control should be greater for a risky firm than a less risky firm with the same exercise price.

Valuing the Synergy Option A synergy option emerges when a buyer has an alternative strategy for the use of the firm's assets. That is, the strategic buyer believes his actions can produce more upside valuation possibilities relative to what is possible under the current regime. Since upside valuation possibilities increase, the strategic buyer can afford to pay an increment above the pure value of control. Let us return to our

earlier example of the sale of the veterinary practice to a strategic buyer who desires to create the dog hotel. The present value of the veterinary practice cash flows is still a $100. Based on the buyer's experience, it will take $50 of investment to create as much as $100 of additional value. However, the expected additional value is only $50, net of the initial investment. If this strategic investment were initiated today, it would have a net present value of zero. But this traditional analysis does not consider the fact that that there is potentially significant upside value to this strategic investment, perhaps as much as an incremental $100 in value. Moreover, the buyer knows that the $50 investment can be postponed to a later time so more of the uncertainty surrounding the possibility of achieving the $100 upside could be resolved. The fact that the strategic investment can get postponed if conditions are not right has value. Like the pure control option, the value of the strategic option is based on the volatility of return and the time to expiration.

Based on past experience and other factors, the buyer expects the synergy strategy to have a volatility of 25%. Keep in mind that this volatility is not the return volatility associated with veterinary practice under old management, but, rather the volatility of asset returns associated with the investment created by the dog hotel strategy. The volatilities will not necessarily be the same because the risk profiles of the cash flows from the business as usual strategy may be very different than the incremental cash flows produced by the dog hotel strategy. For example, if the acquiring firm management has been successful in implementing similar synergistic strategies in the past, then the return volatility will be lower then if the firm were implementing the strategy for the first time. But this does mean that the option is worth less, since a lower risk profile may mean that

the value of expected cash flows is greater relative to the investment, and thus the investment has intrinsic value.[v] Again, these considerations are a function of a known buyer's characteristics and track record.

The final parameter is the time to expiration. Since this is a strategic option, it can be exercised anytime, and hence from this perspective alone it is quite valuable. In finance the period over which the firm is expected to earn rates of return above its cost of capital is called the competitive advantage period. Given that a strategic option is being considered, the time to expiration should coincide with the length of time of the competitive advantage period. As a practical matter the length of time of the competitive advantage varies depending on a multitude of factors, although it is often taken to be five years.[vi]. Based on an exercise price of $50, the expected present value of cash flows of $50, volatility of 25%, a five-year risk free rate of return of 3%, the Black-Scholes model indicates that the strategic option is worth approximately $14.

Putting it All Together Based on Exhibit II, let us assume that the pure control premium has 12 months to expiration and a volatility of 25%. Therefore, the value of pure control is about $11 The value of the synergy option is $14. Thus, the value of the total control premium is $25. In this example, the buyer of the veterinary practice would be willing to pay no more than $125 for the practice or buy the practice at a premium of $25 above the present value of the veterinary practice's standalone cash flows. Clearly, if the buyer has significant negotiating leverage, the premium paid will be lower than 25%. As noted earlier, it appears that this is the case when public firms purchase private firm

targets. Alternatively, if the seller has leverage and the buyer believes that its future is compromised without purchase of the target, then payment in excess of 25% may well be possible. In this case, however, the parameters used to calculate the synergy option would be different and presumably give rise to a larger premium.

VI. A Preliminary Test of the Model

This section reports preliminary results of testing whether there is a relationship between the value of pure control and actual control premiums paid. This test takes two forms. First, our theory suggests that the value of pure control should be no greater than the reported control premium. Hence, we want to test this hypothesis. Second, we want to test whether there is a significant correlation between the estimated values of pure control and the control premiums actually paid. If so, this would indicate, although not prove, that an option-pricing model is a useful first step in estimating the proper size of the control premium in the presence of non-strategic buyers.

The initial sample included 86 firms that were acquired between 1998 and 2001. The data comes from Mergerstat/Shannon Pratt's Control Premium Study.[vii] Of the thousands of transactions reported in this study, we randomly selected 86 acquisitions. For each firm in the sample, we collected end-of-month stock price data for 60 months prior to the 2 month date from which the acquisition premium was calculated. From this data we calculated each stock's volatility as the variance of its monthly returns prior to the 2-month window. The risk free rate was the yield on a government security rate prevailing at the end of the month prior to the 2- month window with a maturity equal to the life of the option. The exercise price was set at the month-end price prior to the 2- month

acquisition window. For each firm the pure control premium was calculated assuming a one year life. The value of the synergy option was calculated as the difference between the reported control premium and the estimated value of the pure control option. Appendix II contains all the data in this study.

Exhibit III summarizes the basic results for the total sample and two sub-samples.

Exhibit III: Control Premium, Value of Pure Control, and Value of Synergy as a Percent of Pre-Announcement Stock Price

	Sample I: Original Sample: 86 Firms			**Sample II: Sample I Less Firms with Negative Control Premiums: 74 Firms in Sample**			**Sample III: Sample II Less Firms with Negative Estimated Synergy Value: 58 Firms in Sample**		
	Average	***Median***	***SD****	***Average***	***Median***	***SD***	***Average***	***Median***	***SD***
Reported Control Premium	47	36	66	56	44	65	66	50	70
Pure Control Premium	22	16	18	21	15	19	17	15	13
Estimated Synergy	26	18	66	36	24	64	49	34	65

* SD = standard deviation

The first sub-sample removes firms with reported negative control premiums. A negative control premium means that the firm was bought for less than the value of its expected cash flows. Without having any additional information about the transaction, this result makes little economic sense. Therefore, we removed these firms from our sample. Sample III, the second sub-sample, removes firms that had negative synergy option values. Sixteen firms fell into this category. Negative synergy option values can arise for at least two reasons. The first reason is that the pure control premium was estimated with sufficient error such that its value exceeded the reported control premium. The error can emerge for a number of reasons. These include the option life being too long, for example

12 months instead of 6, and the estimated volatility being too large. The second reason is that since the acquirer purchased the firm at a discount to the firm's intrinsic value, a negative synergy value implies that the acquiring firm paid less that the value of pure control. Put differently, the seller left money on the table. At this juncture, we have no way of measuring whether the negative difference is due to measurement error or inefficient pricing. However, the fact that these negative differences only occur for 16 firms, or about 20% of the firms in sample II, we expect that they are not the result of measurement error, but, rather, arise because of shrewd bargaining on the part of the buyers. Nevertheless, a more intensive analysis needs to be undertaken before any definitive conclusions can be reached on this point.

The results in Exhibit III are interesting, the drawbacks noted above notwithstanding. First, the value of pure control is less than the reported control premium for 78% of sample II (58/74). Second, the value of pure control is generally far smaller than the value of the synergy option. In 42 out of 58 cases, the synergy option value exceeds the pure control option value, and this result is significantly different than the result obtained by pure chance. In only 4 cases do the differences exceed 10% and, of these, only 2 exceed 20%. This means that in relatively few cases the pure control option value exceeds the value of the synergy option.

This result is consistent with what one would expect. The reason is that acquisitions are generally carried out for strategic reasons, irrespective of whether the combination makes economic sense to stock market investors, and not because the acquirer simply wants to

operate the target in the same way in the future as it has been run in the past. Even in cases where the chief motivation for the acquisition is to end non-economic activities carried out by current management, one would not expect the pure control option to be worth more than the synergy option, the option to end specified activities. Indeed, during the 1980s there were a number of well-publicized takeover attempts whose primary purpose was to change management precisely because it would not respond to stock market pressures to end activities that were wasting corporate resources.[viii]

Overall, Exhibit III indicates that on average the value of pure control is less than the synergy option value. The relative importance of the pure control option declines as we move from sample I to sample III. Sample III indicates that on average, the value of pure control is 17% of the pre-acquisition announcement price, which is about 26% of the acquisition premium. Although not shown, the coefficient of variation of both the pure control and synergy options were calculated for each sample. This metric, measured as the ratio of the standard deviation to the average, indicates that the value of the pure control option varies far less relative to its average than does the value of the synergy option. This is true for all samples, and this result is what one would expect. The reason is that the risks associated with synergy activities are likely to be far more risky than running a standalone business, and the exercise period for implementing the synergy option will certainly be far greater than time to expiration of a pure control option. Where both factors are in play, the synergy option will generally represent the greatest percentage of the reported control premium.

Finally, we estimated a model where the reported control premium is the dependent variable and the pure control option is the independent variable. This exercise was carried out for sample III firms only. Exhibit IV shows the results of this analysis.

Exhibit IV: Relationship Between Reported Control Premium and the Pure Control Option

Multiple R	0.479427062				
R Square	0.229850308				
Adjusted R Square	0.216097634				
Standard Error	0.622338539				
Observations	58				
ANOVA					
	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	6.473085778	6.473086	16.71314	0.00014028
Residual	56	21.68909442	0.387305		
Total	57	28.16218019			
Variables	***Coefficients***	***Standard Error***	***t Stat***	***P-value***	***Lower 95%***
Constant Term	0.219780239	0.135031015	1.627628	0.109218	-0.05071921
Pure Control Option	2.626734985	0.642520922	4.08817	0.00014	1.339611768

The regression model indicates that there is a significant relationship between the values of the pure control option and reported control premiums. The adjusted R^2 is 22% and the coefficient of the pure control option, 2.63, is statistically significant. While these results are promising and support the use of the option-pricing framework when estimating the size of a control premium, much additional research needs to be done. However, these results do lend support to the view that control owners have control options that are valuable apart from the expected cash flows of their firms.

VII. Summary and Conclusions

Most private firm transactions reflect a purchase by a business as usual as opposed to a strategic buyer. In these cases, the control value should only reflect the value of pure control. Implicitly including a synergistic component, for example, by using the median value from published control studies, creates a significant bias in the firm's control value. Second, the value of control is not represented in the expected cash flows of the stand-alone firm. While these expected cash flows represent the expected exercise of control owner options, the value of pure control represents control options not yet exercised. Hence, control has a value in excess of the firm's expected cash flows, and independent of the value that a buyer hopes to create based on expectations of combinatorial synergies.

In this paper we set out a model based on option-pricing theory and offer some preliminary test results that appear to justify using the model in settings where estimating the value of control is highly uncertain. Based on these preliminary findings, for those transactions where the buyer plans to manage the firm in the same way as the seller, the estimated control premium should not exceed the size of the pure control option.

Appendix I: Estimating Private Firm Volatility

Employing the option- pricing model to estimate control premiums requires a measure of return volatility. For private firms, this volatility can be approximated using a principle result from the CAPM shown below.

$$\sigma_i^2 = B_i^2 * \sigma_m^2 + \sigma_{ie}^2 \quad (1)$$

σ^2 is the variance of the volatility of returns for firm i and the market portfolio m respectively. σ_{ie}^2 is non-systematic risk that can be diversified away through portfolio diversification. B_i is the single-factor CAPM beta for firm i.

The expected return for firm i can be estimated from the build-up method.

$$R_i = R_f + Beta_i * RP_m + SP_i + FSP_i \quad (2)$$

R_i is the expected return on the risk free asset. - RP_i , SP_i , and FSP_i are risk premiums that reflect market risk, size risk, and firm-specific risk respectively.

$Beta_i$ in equation 2 is the CAPM beta adjusted for size and firm-specific risk. This beta is defined as $(R_i - R_f)/RP_m$. Equation 2 can now be solved for $(R_i - R_f)/RP_m$.

$$(R_i - R_f)/RP_m = Beta_i + SP_i/RP_m + FSP_i/RP_m \quad (3)$$

The beta calculated using equation (3) is the unlevered beta adjusted for non-systematic risk factors. If the private firm has an optimal capital structure that includes debt, the beta calculated using equation (3) must be further adjusted to reflect this risk using the well-known Hamada relationship. By substituting $Beta_i$ for B_i in equation (1), we can now approximate σ_i^2 under the assumption that σ_{ie}^2 is small or close to zero. Since the two critical non-systematic risk factors determining a firm's risk are now incorporated into the adjusted beta, it is reasonable to assume that diversifiable risk is relatively low.

A first approximation to $Beta_i$ is the unlevered beta for the industry that firm i is part of. The best publicly available source of this data is Ibbotson Associates. Axiom Valuation Solutions has developed unlevered betas for disaggregated industry segments by combining market betas with earnings growth rates for disaggregated industries. These betas are not publicly available, however.

Appendix II: The Data

Target Ticker Symbol	2 Month Premium	Date Announced	Days Prior	Stock Price	Exercise Price (Stock Price)	Volatility (standard deviation of return)	Risk Free Rate	Time Until Option Expiration (in years)	Option Value	Option Value / Stock Price
PDM	0.059	2/1/02	60	31.82	31.82	0.23884339	0.0216	1	3.34	0.105
LEVL	0.811	3/4/99	60	37.4375	37.4375	0.49455878	0.047	1	8.04	0.215
WLL	0.755	11/13/00	60	27.68	27.68	0.24003401	0.0609	1	3.47	0.125
RRI	0.338	7/12/99	60	16.87	16.87	0.16251598	0.0503	1	1.53	0.091
FFWD	0.411	12/17/98	60	13.75	13.75	0.40399322	0.0452	1	2.47	0.180
HOVB	-0.039	1/26/00	60	15.1666667	15.166667	0.16063547	0.0612	1	1.46	0.096
DEX	0.188	7/9/00	60				0.0608	1	#DIV/0!	#DIV/0!
HRBC	-0.146	4/5/00	60	22.4375	22.4375	0.92696737	0.0615	1	8.46	0.377
JPR	0.147	3/4/02	60	22.76	22.76	0.16387286	0.0223	1	1.73	0.076
FCNB	0.853	7/27/00	60	13.3125	13.3125	0.29839351	0.0608	1	1.96	0.147
GNCI	0.471	7/5/99	60	17.75	17.75	0.55828964	0.0503	1	4.26	0.240
IHC	0.518	5/2/02	60	31.9375	31.9375	0.10073903	0.0248	1	1.70	0.053
DI	-0.270	2/26/98	60	41.4375	41.4375	0.17801075	0.0531	1	4.06	0.098
BLCA	0.603	6/28/01	60	23.3	23.3	0.18765806	0.0358	1	2.15	0.092
FSVC	-0.072	8/17/99	60	4.3125	4.3125	0.27786143	0.052	1	0.58	0.135
AQM	1.083	6/14/99	60	3	3	0.32180806	0.051	1	0.45	0.151
GPM	0.290	11/2/00	60	3.5	3.5	0.31779238	0.0609	1	0.54	0.154
DDDP	0.907	1/16/03	60	3.08	3.08	0.1629972	0.0136	1	0.22	0.072
LJLB	0.516	6/8/00	60	8.75	8.75	1.92345225	0.0617	1	5.90	0.674
CBG	0.362	11/13/00	60	11.87	11.87	0.97908969	0.0609	1	4.68	0.395
AXPH	0.146	6/13/01	60	2.76	2.76	0.73686678	0.0358	1	0.83	0.300
CSRV	0.194	9/8/97	60				0.0552	1	#DIV/0!	#DIV/0!
CTYA	0.592	3/5/99	60	31.1875	31.1875	1.09712883	0.0478	1	13.43	0.431
PFFC	0.735	10/2/01	60	7.05	7.05	0.23764061	0.0282	1	0.76	0.108

Target Ticker Symbol	2 Month Premium	Date Announced	Days Prior	Stock Price	Exercise Price (Stock Price)	Volatility (standard deviation of return)	Risk Free Rate	Time Until Option Expiration (in years)	Option Value	Option Value / Stock Price
EACO	0.194	7/24/01	60	1.29	1.29	0.3427552	0.0362	1	0.20	0.152
WFC	0.088	6/8/98	60	35.21	35.21	0.15881825	0.0541	1	3.22	0.091
FSA	0.545	3/14/00	60	49.18	49.18	0.20692364	0.0622	1	5.59	0.114
MTRA	0.499	6/7/99	60	1.25	1.25	0.19860663	0.051	1	0.13	0.105
RATL	1.448	12/6/02	60	5.8	5.8	2.96669103	0.0145	1	5.01	0.863
EXEC	0.413	1/6/99	60	11	11	0.11174124	0.0451	1	0.76	0.069
KSTN	0.363	5/17/00	60	17.75	17.75	0.26845225	0.0633	1	2.43	0.137
OK	0.346	11/20/00	60	0.8875	0.8875	0.67249848	0.0609	1	0.25	0.286
BKC	0.414	7/19/01	60	22.35	22.35	0.2726488	0.0362	1	2.80	0.125
NEWZ	1.018	8/7/01	60	1.17	1.17	0.40310732	0.0347	1	0.20	0.175
CTG	0.063	6/30/99	60	24.06	24.06	0.0756089	0.051	1	1.46	0.061
LUSA	0.711	5/17/99	60	12.125	12.125	0.34272183	0.0485	1	1.91	0.158
NRC	0.170	2/16/99	60	47.625	47.625	0.15963353	0.047	1	4.18	0.088
PATH	0.684	12/9/02	60	13.01	13.01	0.82039827	0.0145	1	4.21	0.323
RELY	0.140	8/30/99	60	29	29	0.32318868	0.052	1	4.41	0.152
PRFC	0.295	6/14/01	60				0.0358	1	#DIV/0!	#DIV/0!
MWFD	0.430	11/12/97	60	21.75	21.75	0.35261924	0.0546	1	3.58	0.164
VLP	0.217	8/29/97	60	13.125	13.125	0.57958798	0.0556	1	3.28	0.250
ARSC	0.663	6/14/01	60	1.19	1.19	0.35566365	0.0358	1	0.19	0.157
NEWI	0.048	7/14/98	60				0.0536	1	#DIV/0!	#DIV/0!
RCHY	0.400	10/1/98	60	6.75	6.75	0.40532011	0.0471	1	1.22	0.181
CMSS	1.386	1/30/01	60	2.25	2.25	0.51928943	0.0481	1	0.50	0.224
EFS	-0.024	11/14/00	60	14.37	14.37	0.40927142	0.0609	1	2.71	0.189
IPSW	0.550	2/27/02	60	13	13	0.54406892	0.0223	1	2.90	0.223
QHGI	0.241	10/19/00	60	12.62	12.62	0.35905307	0.0601	1	2.14	0.169
SBRG	1.006	11/19/01	60	2.435	2.435	0.86088897	0.0218	1	0.83	0.340
ANI	0.441	6/8/98	60				0.0541	1	#DIV/0!	#DIV/0!

Target Ticker Symbol	2 Month Premium	Date Announced	Days Prior	Stock Price	Exercise Price (Stock Price)	Volatility (standard deviation of return)	Risk Free Rate	Time Until Option Expiration (in years)	Option Value	Option Value / Stock Price
OHSL	0.469	8/3/99	60	15	15	0.16731963	0.052	1	1.40	0.093
TRKA	0.241	3/12/99	60	8	8	0.24448183	0.0478	1	0.96	0.120
UWR	0.637	8/23/99	60	21.6875	21.6875	0.15000739	0.052	1	1.89	0.087
RCA	-0.191	2/18/97	60				0.0553	1	#DIV/0!	#DIV/0!
SPN	0.905	6/2/00	60				0.0617	1	#DIV/0!	#DIV/0!
DS	-0.239	1/29/01	60	29.62	29.62	0.45564328	0.0481	1	5.94	0.200
SFAM	0.248	8/12/02	60	4.45	4.45	0.91635736	0.0176	1	1.60	0.359
IFRS	0.957	4/15/02	60	0.69	0.69	0.52754077	0.0248	1	0.15	0.218
PBSC	0.236	7/16/01	60	6.5	6.5	0.09126505	0.0362	1	0.37	0.056
IHF	0.457	6/23/00	60	14.5625	14.5625	16.5652361	0.0617	1	14.56	1.000
CLMT	1.042	4/9/98	60	13.125	13.125	0.15582329	0.0538	1	1.18	0.090
FBCG	0.020	12/15/99	60	19.5	19.5	0.36568767	0.0584	1	3.34	0.171
QDEK	0.040	10/15/98	60	0.40625	0.40625	3.25058746	0.0412	1	0.36	0.898
COHB	0.228	11/24/00	60	17.12	17.12	0.15398899	0.0609	1	1.60	0.094
ASTX	-0.217	10/2/00	60	17.625	17.625	1.05084938	0.0613	1	7.39	0.419
EFBI	0.792	9/25/98	60	28.25	28.25	0.4146943	0.0471	1	5.21	0.185
BKTI	0.578	8/31/01	60	19.125	19.125	0.16801242	0.0347	1	1.61	0.084
GLBN	-0.357	6/15/01	60	3.54465347	3.5446535	1.21826585	0.0358	1	1.66	0.467
FMY	0.316	10/19/98	60	40.375	40.375	0.52161512	0.0412	1	8.98	0.222
HSTC	0.410	5/1/02	60				0.0248	1	#DIV/0!	#DIV/0!
EFIC	0.455	3/20/00	60	1	1	0.43917208	0.0622	1	0.20	0.200
FFOH	0.363	8/16/99	60	12	12	0.31768193	0.052	1	1.80	0.150
AVEI	0.504	11/30/98	60	36	36	1.62545249	0.0453	1	21.35	0.593
ILRN	3.339	1/31/01	60				0.0481	1	#DIV/0!	#DIV/0!
DEPO	0.475	10/19/98	60	1.3125	1.3125	0.33522031	0.0412	1	0.20	0.152
NRL	0.110	3/25/99	60	17.25	17.25	0.55650696	0.0478	1	4.11	0.238
DEFI	0.357	1/8/99	60	6.625	6.625	0.17234235	0.0451	1	0.61	0.091

Target Ticker Symbol	2 Month Premium	Date Announced	Days Prior	Stock Price	Exercise Price (Stock Price)	Volatility (standard deviation of return)	Risk Free Rate	Time Until Option Expiration (in years)	Option Value	Option Value / Stock Price
PZL	0.600	3/25/02	60	13.75	13.75	0.3723601	0.0257	1	2.18	0.159
OEI	-0.455	11/25/98	60	14.37	14.37	1.07519989	0.0453	1	6.07	0.423
HGC	0.294	2/10/99	60	61.0416667	61.041667	0.41183895	0.047	1	11.19	0.183
ERDR	-0.118	11/14/00	60	0.17	0.17	0.34649363	0.0609	1	0.03	0.165
RSND	1.207	5/10/99	60	4.0625	4.0625	0.29725386	0.0485	1	0.57	0.141
SNAP	0.152	11/21/02	60	4.98	4.98	0.31386731	0.0149	1	0.65	0.131
LOIS	0.786	12/19/00	60	2.125	2.125	0.09412457	0.056	1	0.15	0.070
FCBH	5.188	5/22/01	60	0.11	0.11	1.40662754	0.0378	1	0.06	0.527
XLSW	0.300	8/18/99	60	27.75	27.75	0.3547328	0.052	1	4.55	0.164
AMFH	-0.086	8/22/02	60	29.92	29.92	0.17089354	0.0176	1	2.29	0.077
FFA	0.073	3/30/01	60	22.65	22.65	0.11860251	0.043	1	1.59	0.070
OTEC	1.350	2/14/02	60	5.95	5.95	0.23051626	0.0223	1	0.61	0.102
SPYG	0.035	3/26/00	60	37.25	37.25	0.79229181	0.0622	1	12.28	0.330
CKC	0.067	1/12/01	60	10.3	10.3	0.33338745	0.0481	1	1.59	0.154
MBNY	0.301	9/6/00	60	17	17	0.40144052	0.0613	1	3.16	0.186
IGTI	1.590	6/1/00	60	0.625	0.625	0.14718605	0.0633	1	0.06	0.093

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[i] Control Premium Study(Los Angeles: Houlihan Lokey Howard and Zukin, 1995),p.1
[ii] Ibid
[iii] On this point see Pratt, Reilly and Schweihs, *Valuing a Business*, Chapter 14.
[iv] James Ang and Ninon Kohers, " The take-over market for privately-held companies: the US experience", *Cambridge Journal of Economics*, 2001, 25, pp. 723-748. The authors state: "Overall, our results show that, in contrast to acquisitions of publicly traded targets, acquisitions of privately held targets yield substantial gains for both bidder and target firms. Specifically, the event- period, abnormal returns for acquires of privately held targets are significantly positive, regardless of the method of payment used. Thus, takeovers of privately held firms are, on average, perceived too be value enhancing for acquiring firms. Furthermore, private sellers also gain, as the premiums paid to private targets exceed those paid for publicly traded targets in either cash or stock offers." (p.725)
[v] An option has intrinsic value if the expected present value of the cash flows excluding on-going investment requirements, exceeds the present value of the investment requirements. This is termed an "in the money" call option.
[vi] As noted in the text there is nothing magic about five years.
[vii] The Mergerstat/Shannon Pratt's Control Premium Study currently contains approximately 3,450 total transactions; with over 485 deals in business services, over 430 deals on depository institutions, and 138 deals in the communications industry. 51% of the deals in the database have net sales less than $100 million, with the remainder having net sales greater than $100 million.
[viii] In the 1980s, T.Boone Pickens of Mesa Petroleum attempted to acquire Unocal to get access to its oil reserves and to stop the wasting of corporate resources on exploring and drilling for new oil supplies. As it turned out, drilling for oil was a negative NPV investment. T Boone realized that if he had control of Unocal, he could stop the oil drilling activity, which in turn would result in a windfall that in large part would provide the capital to finance the acquisition. As it turned, Unocal management got the message. Unocal's defense in the Mesa tender offer battle resulted in a $2.2 billion (35 percent) gain to shareholders from retrenchment and return of resources to shareholders. Unocal paid out 52 percent of its equity by repurchasing stock with a $4.2 billion debt issue and reduced costs and capital expenditures.